

## Sections 2.1 and 6.2

# Using Deductive Reasoning with Inverses and Contrapositives

**CONDITIONAL STATEMENT ( $p \rightarrow q$ ):** a statement that has two parts, a *hypothesis* and a *conclusion*. The conditional statement can be written in an **IF ( $p$ )  $\rightarrow$  THEN ( $q$ ) STATEMENT**. The “if” part contains the *hypothesis* and the “then” part contains the *conclusion*.

### Identifying the Hypothesis and Conclusion

Identify the parts of the conditional statement (*if* ( $p$ )  $\rightarrow$  *then* ( $q$ ) statement) by underlining the hypothesis and circling the conclusion.

1. If the team wins the semi-final, then the team will play in the championship.
2. If I don't do my homework, then I will not get credit for it.
3. If I am texting on my phone in class, then I will get my phone taken away.

### Writing If – Then Statements

Rewrite each statement as a conditional statement (*if-then* statement).

4. Students who eat Happy Meals are really cool.  
**If a student eats a Happy Meal, then they are really cool.**
  5. Every duck on the pond is hungry.  
**If there is a duck on the pond, then it is hungry.**
  6. I will go running if it does not rain.  
**If it is not raining, then I will go running.**
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**COUNTEREXAMPLE:** an example that shows a conjecture is false.

7. Conjecture: If you are in this class, you are 12 years old.  
**If you are in this class, you are 14 by proof of birth certificate.**
8. Conjecture: If  $x^2 = 16$ , then  $x = 4$ .  
**If  $x^2 = 16$ , then  $x = -4$ .**
9. Conjecture: The product of two positive numbers is always greater than either number.  
 **$0(25) = 0$**

**CONVERSE:** Not just a shoe, but an *if-then* statement formed by switching the hypothesis with the conclusion. Sometimes the converse is true and sometimes it's false. If false, it takes only one counterexample to disprove the statement.

**BICONDITIONAL STATEMENT:** a statement that contains the phrase "*if and only if*" or "*iff*". Writing a biconditional statement is equivalent to writing a conditional statement (*if-then* statement) and its converse. A biconditional statement can either be true or false. If true, both the conditional statement and its converse are true.

### Examples

**Write the converse of each conditional statement. Then identify whether the conditional statement is true or false. If false, provide a counterexample. If true, write a biconditional statement.**

10. Statement: If you go to Troy High School, then you live in Fullerton, CA.

Converse: **If you live in Fullerton, CA, then you go to Troy High School.**

TRUE or **FALSE**

**Counterexample:** **If you live in Fullerton, CA, then you may go to Fullerton High School.**

Biconditional Statement:

11. Statement: If you are awesome, then you are Mr. Lee.

Converse: **If you are Mr. Lee, then you are awesome.**

**TRUE** or FALSE

Counterexample:

**Biconditional Statement:** **You are awesome iff you are Mr. Lee.**

12. Statement: If  $x = -6$ , then  $|x| = 6$ .

Converse: **If  $|x| = 6$ , then  $x = -6$ .**

TRUE or **FALSE**

**Counterexample:** **If  $|x| = 6$ , then  $x = 6$ .**

Biconditional Statement:

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**NEGATION ( $\sim$ ):** making a statement negative (**NOT**).

Write the negation of each statement.

13. Statement:  $\angle A$  an acute angle.

Negation:  **$\angle A$  an NOT acute angle.**

14. Statement: Troy High School is not the best high school.

Negation: **Troy High School is the best high school.**

***INVERSE***: when you negate the hypothesis and conclusion of a conditional statement.

***CONTRAPOSITIVE***: when you negate the hypothesis and conclusion of the converse.

**Write the statement as a conditional statement (if-then statement). Then write the inverse, converse, and the contrapositive of the conditional statement.**

Examples

15. Statement: A figure that has 4 sides is a rectangle.

Conditional Statement: **If a figure has 4 sides, then it is a rectangle.**

Inverse: **If a figure does not have 4 sides, then it is not a rectangle.**

Converse: **If a figure is a rectangle, then it has 4 sides.**

Contrapositive: **If a figure is not a rectangle, then it does not have 4 sides.**

16. Statement: Doing your homework will make you do well in class.

Conditional Statement: **If you do your homework, then you will do well in class.**

Inverse: **If you do not do your homework, then you will not do well in class.**

Converse: **If you do well in class, then you do your homework.**

Contrapositive: **If you do not do well in class, then you do not do your homework.**

17. Statement: Texting on my phone in class will get my phone taken away.

Conditional Statement: **If you are texting on your phone in class, then you will get your phone taken away.**

Inverse: **If you are not texting on your phone in class, then you will not get your phone taken away.**

Converse: **If your phone is taken away, then you are texting on your phone in class.**

Contrapositive: **If your phone is not taken away, then you are not texting on your phone in class.**

**Symbolic Notation**

Hypothesis	Conclusion	Conditional Statement	Negated Hypothesis	Negated Conclusion	Converse	Inverse	Contrapositive
$p$	$q$	$p \rightarrow q$	$\sim p$	$\sim q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$