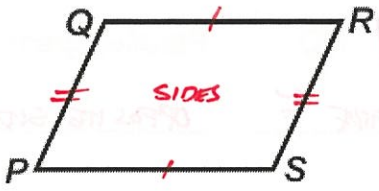
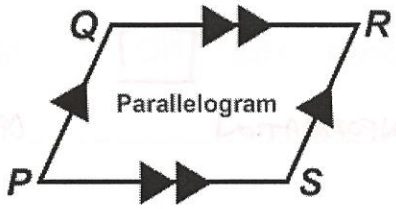
Section 6.3
Proving Quadrilaterals are Parallelograms

Theorem 6.6

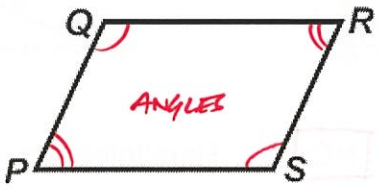
If both pairs of opposite sides of a quadrilateral are congruent,
then the quadrilateral is a parallelogram.

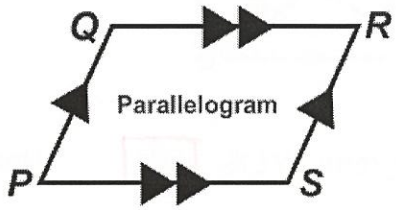
IF 

THEN 

Theorem 6.7

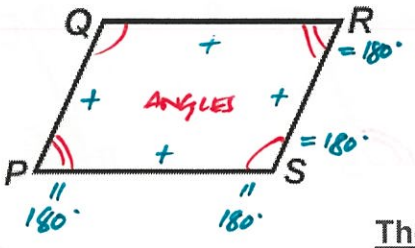
If both pairs of opposite angles of a quadrilateral are congruent,
then the quadrilateral is a parallelogram.

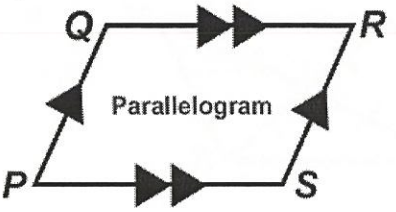
IF 

THEN 

Theorem 6.8

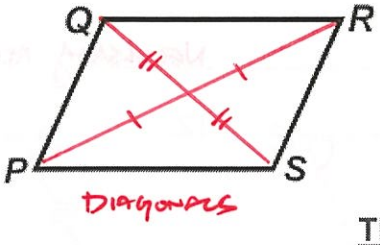
If an angle of a quadrilateral is supplementary to both of it consecutive angles,
then the quadrilateral is a parallelogram.

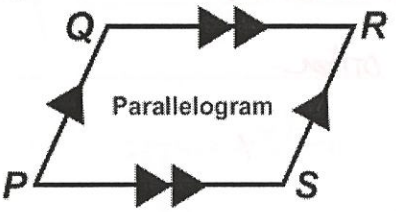
IF 

THEN 

Theorem 6.9

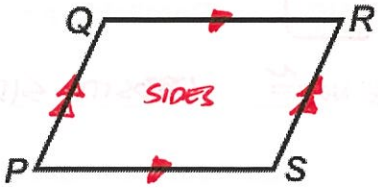
If the diagonals of a quadrilateral bisect each other,
then the quadrilateral is a parallelogram.

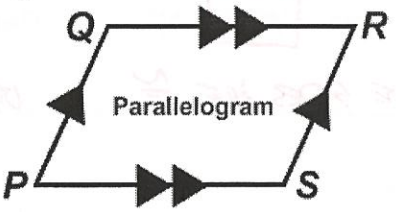
IF 

THEN 

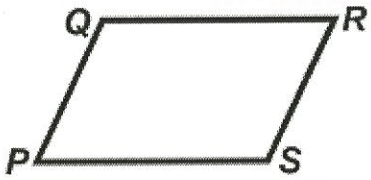
Theorem 6.10

If one pair of opposite sides of a quadrilateral are congruent and parallel,
then the quadrilateral is a parallelogram.

IF 

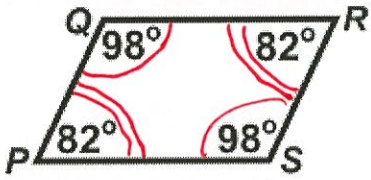
THEN 

Determine whether the following quadrilateral *PQRS* is a parallelogram. Explain why.

1. 

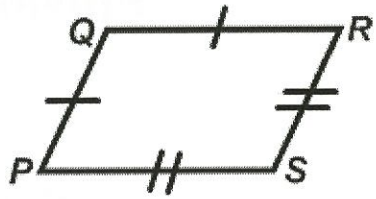
Parallelogram: YES [NO]

NO INFORMATION

2. 

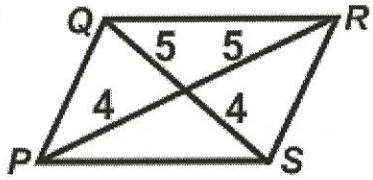
Parallelogram: [YES] NO

OPPOSITE ANGLES ARE $\cong$

3. 

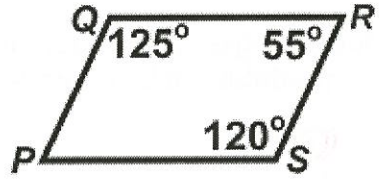
Parallelogram: YES [NO]

OPPOSITE SIDES ARE NOT $\cong$

4. 

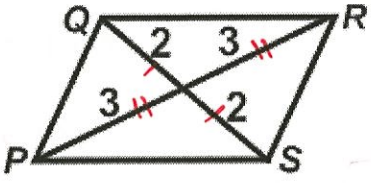
Parallelogram: YES [NO]

DIAGONALS DO NOT BISECT EACH OTHER

5. 

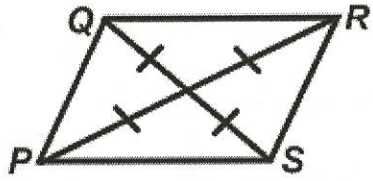
Parallelogram: YES [NO]

OPPOSITE ANGLES ARE NOT $\cong$

6. 

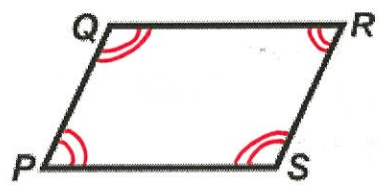
Parallelogram: [YES] NO

DIAGONALS BISECT EACH OTHER

7. 

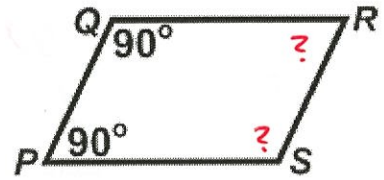
Parallelogram: [YES] NO

DIAGONALS BISECT EACH OTHER

8. 

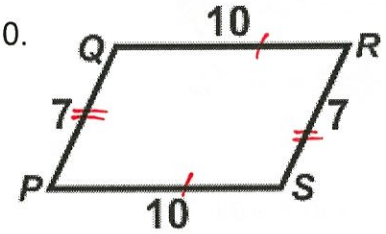
Parallelogram: [YES] NO

OPPOSITE ANGLES ARE $\cong$

9. 

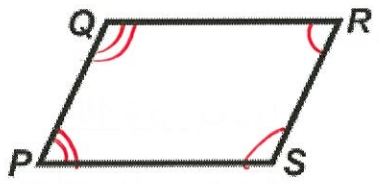
Parallelogram: YES [NO]

CONSECUTIVE INTERIOR DON'T NECESSARY ADD UP TO 180°

10. 

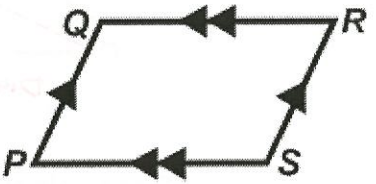
Parallelogram: [YES] NO

OPPOSITE SIDES ARE $\cong$

11. 

Parallelogram: YES [NO]

OPPOSITE ANGLES ARE NOT $\cong$

12. 

Parallelogram: [YES] NO

OPPOSITE SIDES ARE PARALLEL

Using Coordinate Geometry for Parallelograms

If opposite sides of a parallelogram are congruent, then the sides would have the same LENGTHS.

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

If opposite sides of a parallelogram are parallel, then the sides would have the same SLOPES.

$$m=\frac{y_2-y_1}{x_2-x_1}$$

---

13. Prove that $A(2,\ -1)$, $B(1,\ 3)$, $C(6,\ 5)$, and $D(7,\ 1)$ are the vertices of a parallelogram.

METHOD 1 : USING DISTANCE FORMULA

$AB=\sqrt{(1-2)^2+(3-(-1))^2}=\sqrt{17}$

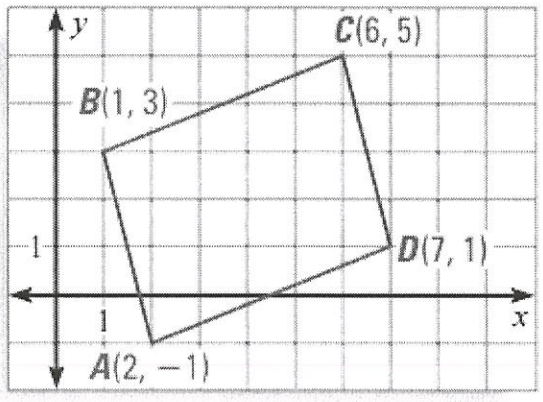
$CD=\sqrt{(7-6)^2+(1-5)^2}=\sqrt{17}$

> OPPOSITE SIDES ARE $\cong$

$BC=\sqrt{(6-1)^2+(5-3)^2}=\sqrt{29}$

$DA=\sqrt{(2-7)^2+(-1-1)^2}=\sqrt{29}$

> OPPOSITE SIDES ARE $\cong$



METHOD 2 : USING SLOPES

$m\overline{AB}=\frac{3-(-1)}{1-2}=-4$

$m\overline{CD}=\frac{1-5}{7-6}=-4$

> OPPOSITE SIDES HAVE SAME SLOPES THEREFORE THEY ARE PARALLEL

$m\overline{BC}=\frac{5-3}{6-1}=\frac{2}{5}$

$m\overline{DA}=\frac{-1-1}{2-7}=\frac{2}{5}$

> OPPOSITE SIDES HAVE SAME SLOPES THEREFORE THEY ARE PARALLEL