

1 POINTS, LINES, PLANES, AND ANGLES



As ancient people studied the heavens, they saw and named many patterns of points, lines, and angles formed by the stars. Although modern observatories use sophisticated observatories and equipment, they still base their calculations on geometric principles that have been known for many centuries.



Some Basic Figures

Objectives

1. Use the term *equidistant*.
2. Use the undefined terms *point*, *line*, and *plane*.
3. Draw representations of points, lines, and planes.
4. Use the terms *collinear*, *coplanar*, and *intersection*.

1-1 A Game and Some Geometry

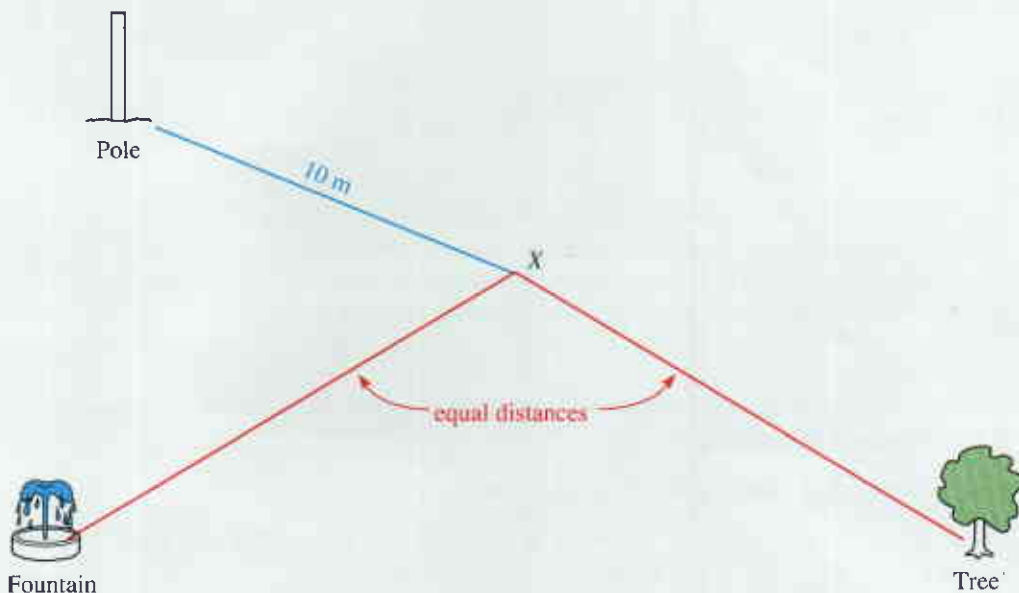
Suppose that you and Pat are partners in a game in which you must locate various clues to win. You are told to pick up your next clue at a point that

1. is as far from the fountain as from the oak tree

and

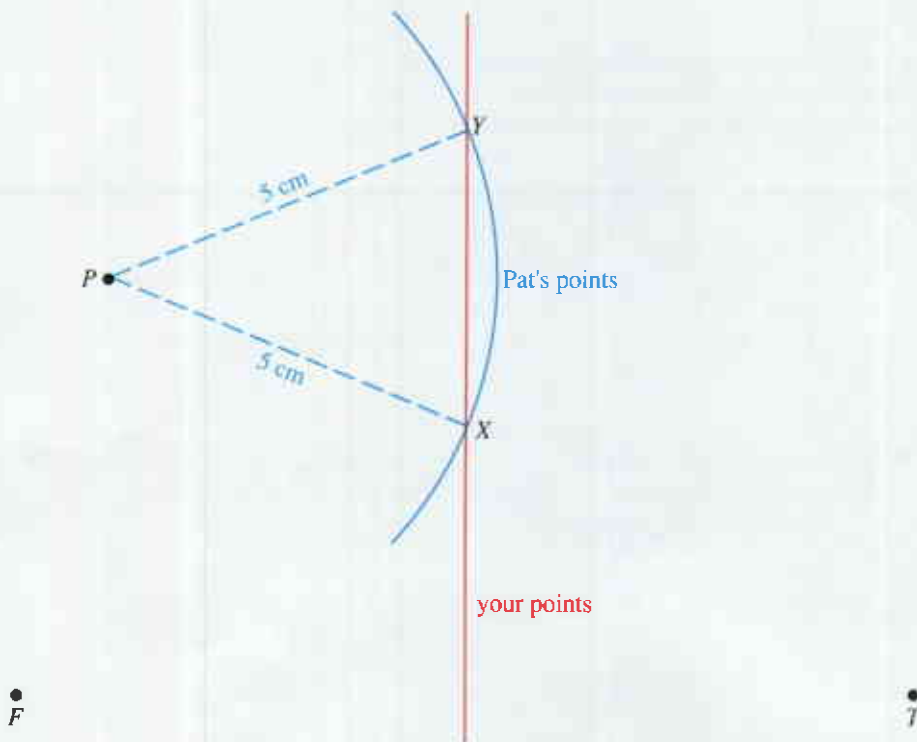
2. is 10 m (meters) from the flag pole.

You locate X , which satisfies both requirements, but grumble because there simply isn't any clue to be found at X .



Then Pat realizes that there may be a different location that satisfies both requirements. (Before reading on, see if you can find another point that meets requirements 1 and 2.)

Suppose that you concentrate on points that satisfy requirement 1 while Pat works on points that meet requirement 2. In the diagram below, 1 cm represents 2 m, so the blue arc shows points that are 10 m from the pole. Each red point is equally distant, or *equidistant*, from F and T . Point Y , as well as X , meets both requirements. You and Pat find your clue at Y and proceed with the game.



The game discussed above involves *points* and *distances*. When you approach the game systematically, you use *lines* and *circles*. Understanding the properties of geometric figures like these is an important part of geometry. The rest of this chapter will deal with the most basic figures of geometry.

Classroom Exercises

For Exercises 1–8 refer to the diagram above.

1. Suppose that the diagram showed, in blue, *all* the points that are 5 cm from P . What geometric figure would the points form?
2. In a more complete diagram, would there be a red point 15 cm from both F and T ? How many such points?
3. It appears as if points P , X , and T might lie on a straight line. Use a ruler or the edge of a sheet of paper to see if they do.

4. It looks as if P might be equidistant from F and X . Is it?
5. Suppose Pat spoke of a line l joining F and T while you thought of a line n joining F and T . Is it better to say that l and n are two different lines, or to say that we have one line with two different names?
6. Point X is equidistant from F and T . Furthermore, point Y is equidistant from F and T . Does that mean that X and Y are equally distant from F ?
7. Suppose you were asked to find a point 5 cm from P , 5 cm from F , and 5 cm from T . Is there such a point?
8. Do you believe there is any point that is equidistant from P , F , and T ?

Written Exercises

- A** 1. Copy and complete the table. Refer to the diagrams on pages 1 and 2.

<i>Distance between</i>	<i>Diagram distance</i>	<i>Ground distance</i>
X and P	$\frac{5}{\quad}$ cm	$\frac{10}{\quad}$ m
X and F	$\frac{7}{\quad}$ cm	$\frac{?}{\quad}$ m
X and T	$\frac{?}{\quad}$ cm	$\frac{?}{\quad}$ m
Y and F	$\frac{?}{\quad}$ cm	$\frac{19}{\quad}$ m
F and T	$\frac{12}{\quad}$ cm	$\frac{?}{\quad}$ m

For Exercises 2–4 use a centimeter ruler. If you don't have a centimeter ruler, you may use the centimeter ruler shown below as a guide. Either open your compass to the appropriate distance or mark the appropriate distance on the edge of a sheet of paper.

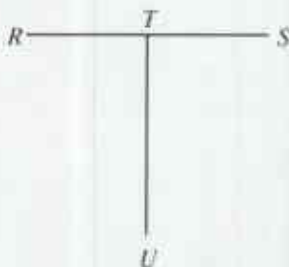


2. Copy the points F , T , and P from the diagram on page 2. If you lay your paper over the page, you can see through the paper well enough to get the points.
 - a. Draw a line to indicate all points equidistant from F and T .
 - b. Draw a circle to indicate points 6 cm from P . If you don't have a compass, draw as well as you can freehand.
 - c. How many points are equidistant from F and T , and are also 6 cm from P ?
3. Repeat Exercise 2, but use 2 cm instead of 6 cm.
4. There is a distance you could use in parts (b) and (c) of Exercise 2 that would lead to the answer *one point* in part (c). Estimate that distance.

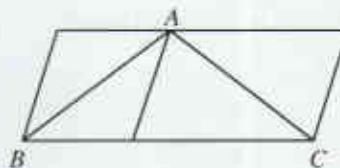
The spoon in the photograph appears to be broken because light rays bend as they go from air to water. As in the photograph, your eyes may mislead you in some of the exercises that follow, but you are asked to make estimates. You may want to check your estimates by measuring.



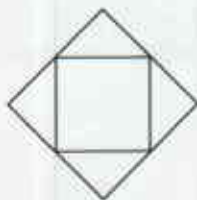
5. Which is greater, the distance from R to S or the distance from T to U ?



6. Which is greater, the distance from A to B or the distance from A to C ?



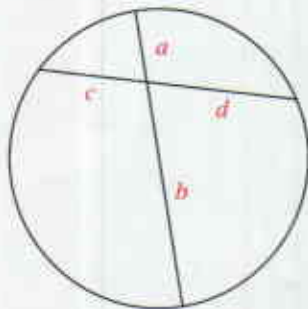
- B** 7. How does the area of the outer square compare with the area of the inner square?



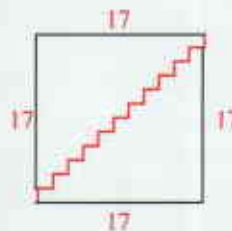
8. Compare the areas of the red and blue regions. (Area of circle = πr^2 .)



9. In the diagram a , b , c , and d are lengths. Which is greater, the product ab or the product cd ?



10. A path between opposite vertices of the square is made up of hundreds of horizontal and vertical segments. (The diagram shows a simplified version.) What is the best approximation to the length of the path—24, 34, 44, or more than 44?



1-2 Points, Lines, and Planes

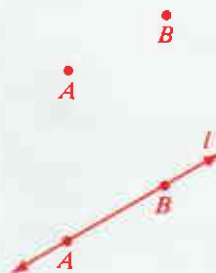
When you look at a color television picture, how many different colors do you see? Actually, the picture is made up of just three colors—red, green, and blue. Most color television screens are covered with more than 300,000 colored dots, as shown in the enlarged diagram below. Each dot glows when it is struck by an electron beam. Since the dots are so small, and so close together, your eye sees a whole image rather than individual dots.



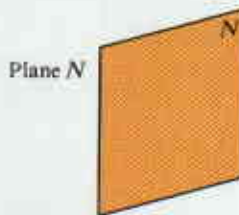
Each dot on a television screen suggests the simplest figure studied in geometry—a *point*. Although a point doesn't have any size, it is often represented by a dot that does have some size. You usually name points by capital letters. Points A and B are pictured at the right.

All geometric figures consist of points. One familiar geometric figure is a *line*, which extends in two directions without ending. Although a picture of a line has some thickness, the line itself has no thickness.

Often a line is referred to by a single lower-case letter, such as *line l* . If you know that a line contains the points A and B , you can also call it *line AB* (denoted \overleftrightarrow{AB}) or *line BA* (\overleftrightarrow{BA}).



A geometric *plane* is suggested by a floor, wall, or table top. Unlike a table top, a plane extends without ending and has no thickness. Although a plane has no edges, we usually picture a plane by drawing a four-sided figure as shown below. We often label a plane with a capital letter.



In geometry, the terms *point*, *line*, and *plane* are accepted as intuitive ideas and are not defined. These *undefined terms* are then used in the definitions of other terms, such as those at the top of the next page.

Space is the set of all points. **Collinear points** are points all in one line.



Collinear points

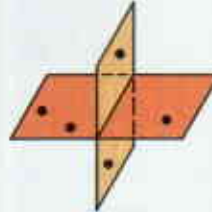


Noncollinear points

Coplanar points are points all in one plane.



Coplanar points



Noncoplanar points

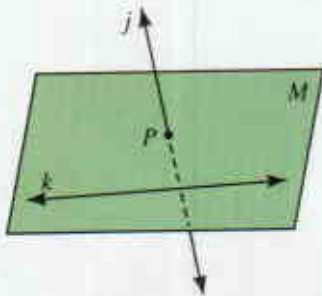
Some expressions commonly used to describe relationships between points, lines, and planes follow. In these expressions, *intersects* means “meets” or “cuts.” The **intersection** of two figures is the set of points that are in both figures. Dashes in the diagrams indicate parts hidden from view in figures in space.



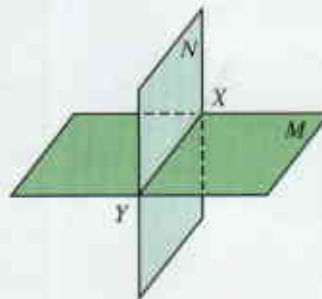
A is in l , or A is on l .
 l contains A .
 l passes through A .



l and h intersect in O .
 l and h intersect at O .
 O is the intersection of l and h .



k and P are in M .
 M contains k and P .
 j intersects M at P .
 P is the intersection of j and M .



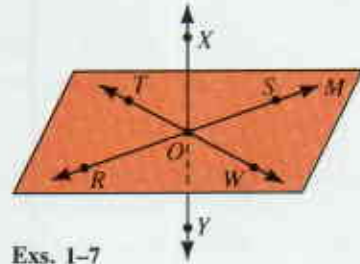
M and N intersect in \overleftrightarrow{XY} .
 \overleftrightarrow{XY} is the intersection of M and N .
 \overleftrightarrow{XY} is in M and N .
 M and N contain \overleftrightarrow{XY} .

In this book, whenever we refer, for example, to “two points” or “three lines,” we will mean *different* points or lines (or other geometric figures).

Classroom Exercises

Classify each statement as true or false.

- \overleftrightarrow{XY} intersects plane M at point O .
- Plane M intersects \overleftrightarrow{XY} in more than one point.
- T , O , and R are collinear.
- X , O , and Y are collinear.
- R , O , S , and W are coplanar.
- R , S , T , and X are coplanar.
- R , X , O , and Y are coplanar.
- Does a plane have edges?
- Can a given point be in two lines? in ten lines?
- Can a given line be in two planes? in ten planes?



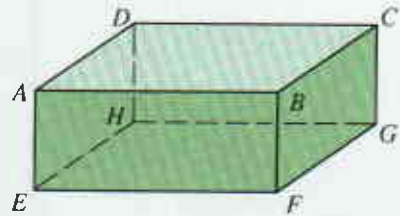
Exs. 1-7

Name a fourth point that is in the same plane as the given points.

11. A, B, C 12. E, F, H 13. D, C, H
 14. A, D, E 15. B, E, F 16. B, G, C

The plane that contains the top of the box can be called plane $ABCD$.

- Are there any points in \overleftrightarrow{CG} besides C and G ?
- Are there more than four points in plane $ABCD$?
- Name the intersection of planes $ABFE$ and $BCGF$.
- Name two planes that do not intersect.

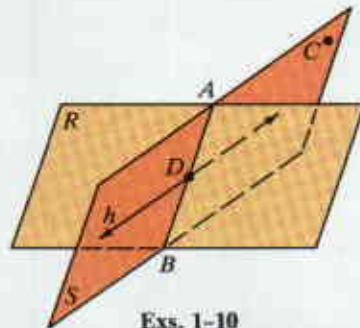


Exs. 11-20

Written Exercises

Classify each statement as true or false.

- A**
- \overleftrightarrow{AB} is in plane R .
 - S contains \overleftrightarrow{AB} .
 - R and S contain D .
 - D is on line h .
 - h is in S .
 - h is in R .
 - Plane R intersects plane S in \overleftrightarrow{AB} .
 - Point C is in R and S .
 - A , B , and C are collinear.
 - A , B , C , and D are coplanar.

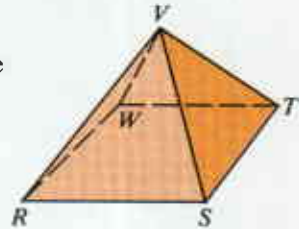


Exs. 1-10

11. Make a sketch showing four coplanar points such that three, but not four, of them are collinear.
12. Make a sketch showing four points that are not coplanar.

A plane can be named by three or more noncollinear points it contains. In Chapter 12 you will study *pyramids* like the one shown at the right below.

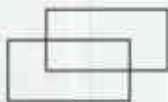
13. Name five planes that contain sides of the pyramid shown.
14. Of the five planes containing sides of the pyramid, are there any that do not intersect?
15. Name three lines that intersect at point R .
16. Name two planes that intersect in \overleftrightarrow{ST} .
17. Name three planes that intersect at point S .
18. Name a line and a plane that intersect in a point.



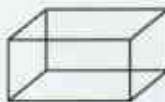
Exs. 13–18

Follow the steps shown to draw the figure named.

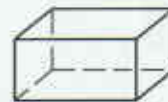
19. a rectangular solid or box



Step 1

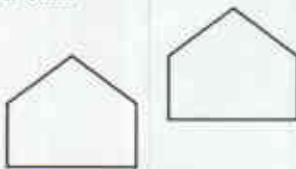


Step 2

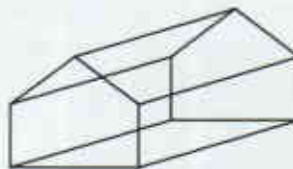


Step 3

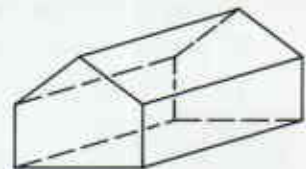
20. a barn



Step 1



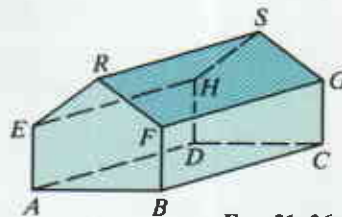
Step 2



Step 3

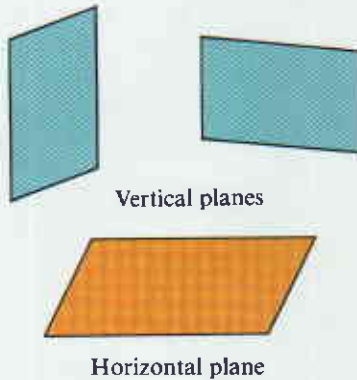
Note: After drawing more figures in space, you will probably be able to go directly from Step 1 to Step 3.

21. Name two planes that intersect in \overleftrightarrow{FG} .
22. Name three lines that intersect at point E .
23. Name three planes that intersect at point B .
24.
 - a. Are points A , D , and C collinear?
 - b. Are points A , D , and C coplanar?
25.
 - a. Are points R , S , G , and F coplanar?
 - b. Are points R , S , G , and C coplanar?
26.
 - a. Name two planes that do not intersect.
 - b. Name two other planes that do not intersect.



Exs. 21–26

You can think of the ceiling and floor of a room as parts of *horizontal planes*. The walls are parts of *vertical planes*. Vertical planes are represented by figures like those shown in which two sides are vertical. A horizontal plane is represented by a figure like that shown, with two sides horizontal and no sides vertical.



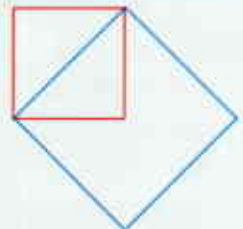
- B** 27. Can two horizontal planes intersect?
 28. a. Can two vertical planes intersect?
 b. Suppose a line is known to be in a vertical plane. Does the line have to be a vertical line?

Sketch and label the figures described. Use dashes for hidden parts.

29. Vertical line l intersects a horizontal plane M at point O .
 30. Horizontal plane P contains two lines k and n that intersect at point A .
 31. Horizontal plane Q and vertical plane N intersect.
 32. Vertical planes X and Y intersect in \overleftrightarrow{AB} .
 33. Point P is not in plane N . Three lines through point P intersect N in points A , B , and C .
C 34. Three vertical planes intersect in a line.
 35. A vertical plane intersects two horizontal planes in lines l and n .
 36. Three planes intersect in a point.

Challenge

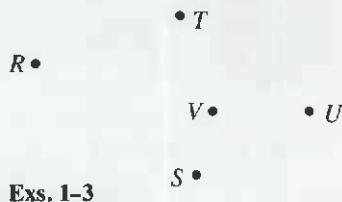
If the area of the red square is 1 square unit, what is the area of the blue square? Give a convincing argument.



Self-Test 1

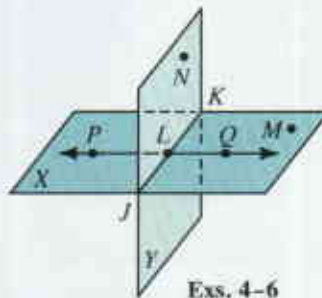
Name the point that appears to satisfy the description.

1. Equidistant from R and S
2. Equidistant from S and U
3. Equidistant from U and T



Classify each statement as true or false.

4. Plane Y and \overleftrightarrow{PQ} intersect in point L .
5. Points J , K , L , and N are coplanar.
6. Points J , L , and Q are collinear.
7. Draw a vertical plane Z intersecting a horizontal line l in a point T .



Algebra Review: Linear Equations

Find the value of the variable.

- | | | |
|----------------------------------|---------------------------------|--------------------------|
| 1. $c + 5 = 12$ | 2. $8 + c = 13$ | 3. $c - 5 = 12$ |
| 4. $7 - z = 13$ | 5. $15 - z = 0$ | 6. $4x = 28$ |
| 7. $3x = 15$ | 8. $7x = -35$ | 9. $-5x = -5$ |
| 10. $\frac{1}{3}a = 2$ | 11. $\frac{3}{4}a = 9$ | 12. $\frac{4}{5}a = -20$ |
| 13. $-2b = 6$ | 14. $-3b = -9$ | 15. $-9b = 2$ |
| 16. $42 = 6k$ | 17. $5 = 10k$ | 18. $-16 = -4k$ |
| 19. $12 = \frac{e}{2}$ | 20. $-9 = \frac{e}{3}$ | 21. $5 = -\frac{e}{3}$ |
| 22. $2p + 5 = 13$ | 23. $3p - 5 = 13$ | 24. $4p + 2 = 22$ |
| 25. $60 = 6t + 12$ | 26. $12 = 3r - 9$ | 27. $55 = 7s - 8$ |
| 28. $8x + 2x = 90$ | 29. $8x - 2x = 90$ | 30. $x + 9x = 5$ |
| 31. $(2g - 15) + g = 9$ | 32. $3u + (u - 2) = 10$ | 33. $(w - 20) + 5w = 28$ |
| 34. $3x = 2x - 17$ | 35. $5y = 3y + 26$ | 36. $7z = 180 - 2z$ |
| 37. $12 + 3b = 2 + 5b$ | 38. $4c + 23 = 9c - 7$ | |
| 39. $7h + (90 - h) = 210$ | 40. $5x + (180 - x) = 300$ | |
| 41. $(4f + 5) + (5f + 40) = 180$ | 42. $(3g - 4) + (4g + 10) = 90$ | |
| 43. $2(4d + 4) = d + 1$ | 44. $2(d + 5) = 3(d - 2)$ | |
| 45. $180 - x = 3(90 - x)$ | 46. $3(180 - y) = 2(90 - y)$ | |

Definitions and Postulates

Objectives

1. Use symbols for lines, segments, rays, and distances; find distances.
2. Name angles and find their measures.
3. State and use the Segment Addition Postulate and the Angle Addition Postulate.
4. Recognize what you can conclude from a diagram.
5. Use postulates and theorems relating points, lines, and planes.

1-3 Segments, Rays, and Distance

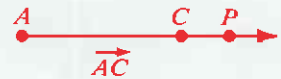
In the diagram, point B is *between* points A and C . Note that B must lie on \overleftrightarrow{AC} .



Segment AC , denoted \overline{AC} , consists of points A and C and all points that are between A and C . Points A and C are called the *endpoints* of \overline{AC} .



Ray AC , denoted \overrightarrow{AC} , consists of \overline{AC} and all other points P such that C is between A and P . The *endpoint* of \overrightarrow{AC} is A , the point named first.



\overrightarrow{SR} and \overrightarrow{ST} are called **opposite rays** if S is between R and T .



The hands of the clock shown suggest opposite rays.



On a *number line* every point is paired with a number and every number is paired with a point. In the diagram, point J is paired with -3 , the *coordinate* of J .



The **length** of \overline{MJ} , denoted by MJ , is the distance between point M and point J . You can find the length of a segment on a number line by subtracting the coordinates of its endpoints:

$$MJ = 4 - (-3) = 7$$

Notice that since a length must be a positive number, you subtract the lesser coordinate from the greater one. Actually, the distance between two points is the absolute value of the difference of their coordinates. When you use absolute value, the order in which you subtract coordinates doesn't matter.



$$\begin{array}{l}
 JL = |-3 - 2| = |-5| = 5 \qquad PQ = |x - y| \\
 \text{or} \\
 JL = |2 - (-3)| = |5| = 5 \qquad PQ = |y - x|
 \end{array}$$

There are many different ways to pair the points on a line with numbers. For example, the red coordinates shown below would give distances in centimeters. The blue coordinates would give distances in inches.



Once you have chosen a unit of measure, the distance between any two points will be the same no matter where you place the coordinate 0. For example, the black coordinates below show another way of assigning coordinates to points on the line so that distances will be measured in inches.



Using number lines involves the following basic assumptions. Statements such as these that are accepted without proof are called **postulates** or **axioms**. Notice that the Ruler Postulate below allows you to measure distances using centimeters or inches or any other convenient unit. But once a unit of measure has been chosen for a particular problem, you must use that unit throughout the problem.

Postulate 1 *Ruler Postulate*

1. The points on a line can be paired with the real numbers in such a way that any two points can have coordinates 0 and 1.
2. Once a coordinate system has been chosen in this way, the distance between any two points equals the absolute value of the difference of their coordinates.

Postulate 2 *Segment Addition Postulate*

If B is between A and C , then

$$AB + BC = AC.$$

Example B is between A and C , with $AB = x$, $BC = x + 6$, and $AC = 24$. Find:

a. the value of x

b. BC

Solution a. $AB + BC = AC$

$$x + (x + 6) = 24$$

$$2x + 6 = 24$$

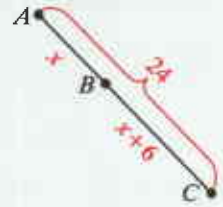
$$2x = 18$$

$$x = 9$$

b. $BC = x + 6$

$$= 9 + 6$$

$$= 15$$



In geometry two objects that have the same size and shape are called **congruent**. For many geometric figures we can give a more precise definition of what it means to be congruent. For example, we will define congruent segments in this section, congruent angles in the next section, congruent triangles in Chapter 4, and congruent circles and arcs in Chapter 9.

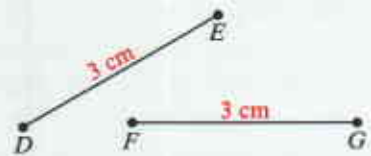
Congruent segments are segments that have equal lengths. To indicate that \overline{DE} and \overline{FG} have equal lengths, you write

$$\overline{DE} = \overline{FG}.$$

To indicate that \overline{DE} and \overline{FG} are congruent, you write

$$\overline{DE} \cong \overline{FG}$$

(read “ \overline{DE} is congruent to \overline{FG} ”). The definition tells us that the two statements are equivalent. We will use them interchangeably.



The **midpoint of a segment** is the point that divides the segment into two congruent segments. In the diagram

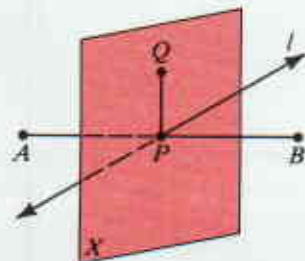
$$\overline{AP} = \overline{PB},$$

$$\overline{AP} \cong \overline{PB},$$

and P is the midpoint of \overline{AB} .

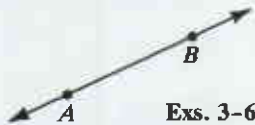


A **bisector of a segment** is a line, segment, ray, or plane that intersects the segment at its midpoint. Line l is a bisector of \overline{AB} . \overline{PQ} and plane X also bisect \overline{AB} .



Classroom Exercises

- Does the symbol represent a line, segment, ray, or length?
 - \overline{PQ}
 - \overrightarrow{PQ}
 - \overleftrightarrow{PQ}
 - PQ
- How many endpoints does a segment have? a ray? a line?
- Is \overline{AB} the same as \overline{BA} ?
- Is \overrightarrow{AB} the same as \overrightarrow{BA} ?
- Is \overleftrightarrow{AB} the same as \overleftrightarrow{BA} ?
- Is AB the same as BA ?



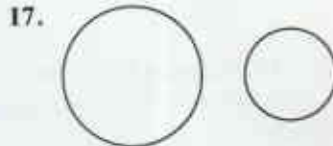
- What is the coordinate of P ? of R ?
- Name the point with coordinate 2.
- Find each distance: a. RS b. RQ c. PT
- Name three segments congruent to \overline{PQ} .



- Name the ray opposite to \overrightarrow{SP} .
- Name the midpoint of \overline{PT} .
- What number is halfway between 1 and 2?
 - What is the coordinate of the midpoint of \overline{ST} ?
- Could you list all the numbers between 1 and 2?
 - Is there a point on the number line for every number between 1 and 2?
 - Is there any limit to the number of points between S and T ?

Exs. 7-14

State whether the figures *appear* to be congruent (that is, appear to have the same size and shape).



- Draw two points P and Q on a sheet of paper. Fold the paper so that fold line f contains both P and Q . Unfold the paper. Now fold so that P falls on Q . Call the second fold g . Lay the paper flat and label the intersection of f and g as point X . How are points P , Q , and X related? Explain.
- If $AB = BC$, must point B be the midpoint of \overline{AC} ? Explain.

The given numbers are the coordinates of two points on a number line. State the distance between the points.

- 2 and 6
- 2 and -6
- 2 and -6
- 7 and -1

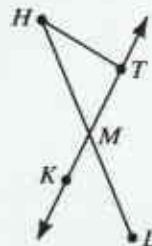
Written Exercises

The numbers given are the coordinates of two points on a number line. State the distance between the points.

- A** 1. -6 and 9 2. -3 and -17 3. -1.2 and -5.7 4. -2.5 and 4.6

In the diagram, \overline{HL} and \overleftrightarrow{KT} intersect at the midpoint of \overline{HL} . Classify each statement as true or false.

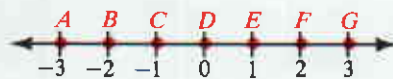
- | | |
|--|--|
| 5. $\overline{LM} \cong \overline{MH}$ | 6. KM must equal MT . |
| 7. \overline{MT} bisects \overline{LH} . | 8. \overleftrightarrow{KT} is a bisector of \overline{LH} . |
| 9. \overrightarrow{MT} and \overrightarrow{TM} are opposite rays. | 10. \overrightarrow{MT} and \overrightarrow{MK} are opposite rays. |
| 11. \overline{LH} is the same as \overline{HL} . | 12. \overleftrightarrow{KT} is the same as \overleftrightarrow{KM} . |
| 13. \overleftrightarrow{KT} is the same as \overleftrightarrow{KM} . | 14. \overline{KT} is the same as \overline{KM} . |
| 15. $HM + ML = HL$ | 16. $TM + MH = TH$ |
| 17. T is between H and M . | 18. M is between K and T . |



Exs. 5-18

Name each of the following.

19. The point on \overrightarrow{DA} whose distance from D is 2
20. The point on \overrightarrow{DG} whose distance from D is 2
21. Two points whose distance from E is 2
22. The ray opposite to \overrightarrow{BE}
23. The midpoint of \overline{BF}
24. The coordinate of the midpoint of \overline{BD}
25. The coordinate of the midpoint of \overline{AE}
26. A segment congruent to \overline{AF}



Exs. 19-26

In Exercises 27-30 draw \overline{CD} and \overline{RS} so that the conditions are satisfied.

27. \overline{CD} and \overline{RS} intersect, but neither segment bisects the other.
28. \overline{CD} and \overline{RS} bisect each other.
29. \overline{CD} bisects \overline{RS} , but \overline{RS} does not bisect \overline{CD} .
30. \overline{CD} and \overline{RS} do not intersect, but \overleftrightarrow{CD} and \overleftrightarrow{RS} do intersect.

- B** 31. In the diagram, $\overline{PR} \cong \overline{RT}$, S is the midpoint of \overline{RT} , $QR = 4$, and $ST = 5$. Complete.
- | | |
|-------------|-------------|
| a. $RS = ?$ | b. $RT = ?$ |
| c. $PR = ?$ | d. $PQ = ?$ |



32. In the diagram, X is the midpoint of \overline{VZ} , $VW = 5$, and $WY = 20$. Find the coordinates of W , X , and Y .



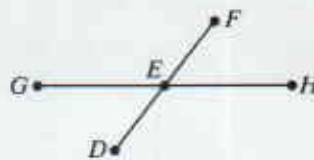
E is the midpoint of \overline{DF} . Find the value of x .

33. $DE = 5x + 3, EF = 33$

34. $DE = 45, EF = 5x - 10$

35. $DE = 3x, EF = x + 6$

36. $DE = 2x - 3, EF = 5x - 24$



Exs. 33-40

Find the value of y .

37. $GE = y, EH = y - 1, GH = 11$

38. $GE = 3y, GH = 7y - 4, EH = 24$

Find the value of z . Then find GE and EH and state whether E is the midpoint of \overline{GH} .

39. $GE = z + 2, GH = 20, EH = 2z - 6$

40. $GH = z + 6, EH = 2z - 4, GE = z$

Name the graph of the given equation or inequality.

Example a. $x \geq 2$ b. $4 \leq x \leq 6$

Solution a. \overrightarrow{NT} b. \overline{TY}



Exs. 41-45

41. $-2 \leq x \leq 2$ 42. $x \leq 0$ 43. $|x| \leq 4$ 44. $|x| \geq 0$ 45. $|x| = 0$

In Exercises 46 and 47 draw a diagram to illustrate your answer.

46. a. On \overrightarrow{AB} , how many points are there whose distance from point A is 3 cm?

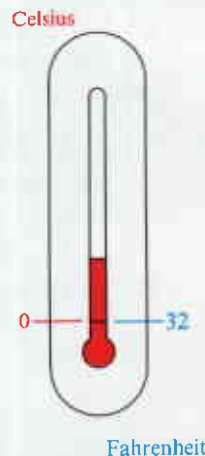
b. On \overleftrightarrow{AB} , how many points are there whose distance from point A is 3 cm?

C 47. On \overrightarrow{AB} , how many points are there whose distance from point B is 3 cm?

48. The Ruler Postulate suggests that there are many ways to assign coordinates to a line. The Fahrenheit and Celsius temperature scales on a thermometer indicate two such ways of assigning coordinates. A Fahrenheit temperature of 32° corresponds to a Celsius temperature of 0° . The formula, or rule, for converting a Fahrenheit temperature F into a Celsius temperature C is

$$C = \frac{5}{9}(F - 32).$$

- What Celsius temperatures correspond to Fahrenheit temperatures of 212° and 98.6° ?
- Solve the equation above for F to obtain a rule for converting Celsius temperatures to Fahrenheit temperatures.
- What Fahrenheit temperatures correspond to Celsius temperatures of -40° and 2000° ?



1-4 Angles

An **angle** (\angle) is the figure formed by two rays that have the same endpoint. The two rays are called the **sides** of the angle, and their common endpoint is the **vertex** of the angle.

The sides of the angle shown are \overrightarrow{BA} and \overrightarrow{BC} . The vertex is point B . The angle can be called $\angle B$, $\angle ABC$, $\angle CBA$, or $\angle 1$. If three letters are used to name an angle, the middle letter must name the vertex.

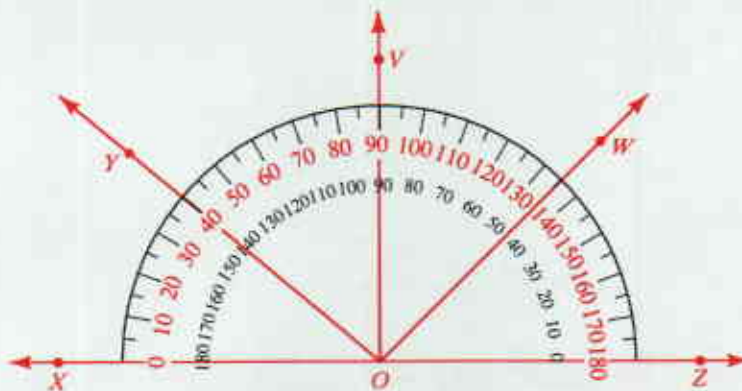
When you talk about this $\angle B$, everyone knows what angle you mean. But if you tried to talk about $\angle E$ in the diagram at the right, people wouldn't know which angle you meant. There are three angles with vertex E . To name any particular one of them you need to use either three letters or a number.

$\angle 2$ could also be called $\angle RES$ or $\angle SER$.

$\angle 3$ could also be called $\angle SET$ or $\angle TES$.

$\angle RET$ could also be called $\angle TER$.

You can use a protractor like the one shown below to find the *measure in degrees* of an angle. Although angles are sometimes measured in other units, this book will always use degree measure. Using the outer (red) scale of the protractor, you can see that $\angle XOY$ is a 40° angle. You can indicate that the (degree) measure of $\angle XOY$ is 40 by writing $m\angle XOY = 40$.



Using the inner scale of the protractor, you find that:

$$m\angle YOZ = 140 \quad m\angle WOZ = 45 \quad m\angle YOW = 140 - 45 = 95$$

Angles are classified according to their measures.

Acute angle: Measure between 0 and 90

Right angle: Measure 90

Obtuse angle: Measure between 90 and 180

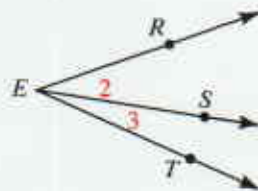
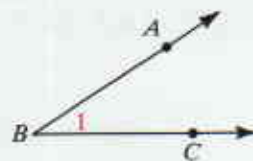
Straight angle: Measure 180

Examples: $\angle XOY$ and $\angle VOW$

Examples: $\angle XOY$ and $\angle YOZ$

Examples: $\angle XOW$ and $\angle YOW$

Example: $\angle XOZ$

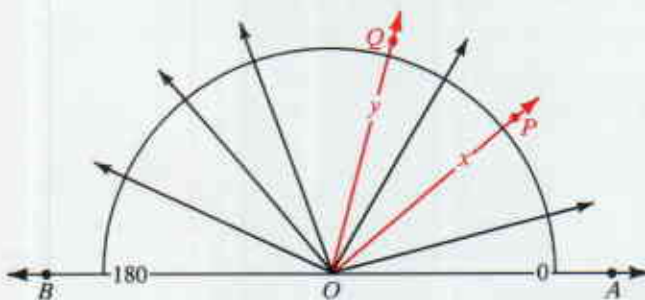


The two angle postulates below are very much like the Ruler Postulate and the Segment Addition Postulate on page 12.

Postulate 3 Protractor Postulate

On \overleftrightarrow{AB} in a given plane, choose any point O between A and B . Consider \overrightarrow{OA} and \overrightarrow{OB} and all the rays that can be drawn from O on one side of \overleftrightarrow{AB} . These rays can be paired with the real numbers from 0 to 180 in such a way that:

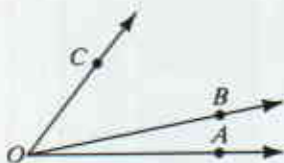
- \overrightarrow{OA} is paired with 0, and \overrightarrow{OB} with 180.
- If \overrightarrow{OP} is paired with x , and \overrightarrow{OQ} with y , then $m\angle POQ = |x - y|$.



Postulate 4 Angle Addition Postulate

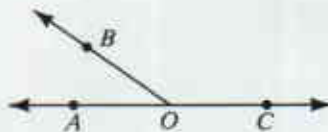
If point B lies in the interior of $\angle AOC$, then

$$m\angle AOB + m\angle BOC = m\angle AOC.$$



If $\angle AOC$ is a straight angle and B is any point not on \overleftrightarrow{AC} , then

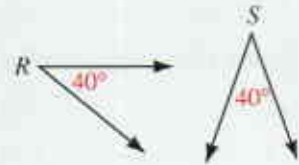
$$m\angle AOB + m\angle BOC = 180.$$



Congruent angles are angles that have equal measures. Since $\angle R$ and $\angle S$ both have measure 40, you can write

$$m\angle R = m\angle S \text{ or } \angle R \cong \angle S.$$

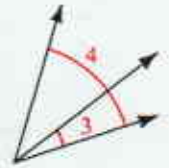
The definition of congruent angles tells us that these two statements are equivalent. We will use them interchangeably.



Adjacent angles (adj. \triangle) are two angles in a plane that have a common vertex and a common side but no common interior points.

$\angle 1$ and $\angle 2$ are adjacent angles.

$\angle 3$ and $\angle 4$ are not adjacent angles.



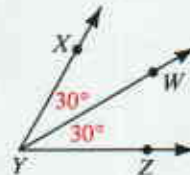
The **bisector of an angle** is the ray that divides the angle into two congruent adjacent angles. In the diagram,

$$m\angle XYW = m\angle WYZ,$$

$$\angle XYW \cong \angle WYZ,$$

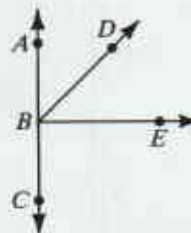
and

$$\overrightarrow{YW} \text{ bisects } \angle XYZ.$$

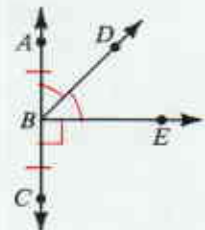


There are certain things that you can conclude from a diagram and others that you can't. The following are things you can conclude from the diagram shown below.

- All points shown are coplanar.
- \overrightarrow{AB} , \overrightarrow{BD} , and \overrightarrow{BE} intersect at B .
- A , B , and C are collinear.
- B is between A and C .
- $\angle ABC$ is a straight angle.
- D is in the interior of $\angle ABE$.
- $\angle ABD$ and $\angle DBE$ are adjacent angles.



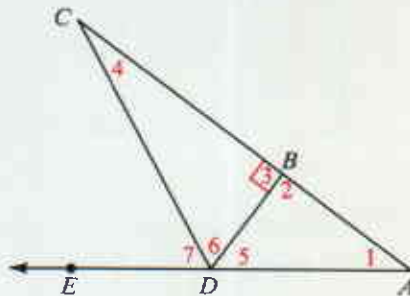
The diagram above does *not* tell you that $\overline{AB} \cong \overline{BC}$, that $\angle ABD \cong \angle DBE$, or that $\angle CBE$ is a right angle. These three new pieces of information can be indicated in a diagram by using marks as shown at the right. Note that a small square is used to indicate a right angle (rt. \angle).



Classroom Exercises

Name the vertex and the sides of the given angle.

- $\angle 4$
- $\angle 1$
- $\angle 6$
- Name all angles adjacent to $\angle 6$.
- Name three angles that have B as the vertex.
- How many angles have D as the vertex?



Exs. 1-16

State whether the angle appears to be acute, right, obtuse, or straight. Then estimate its measure.

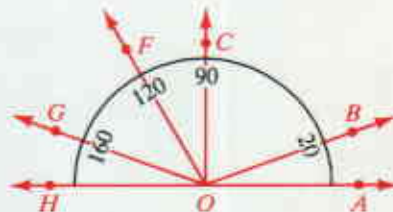
- $\angle 1$
- $\angle 2$
- $\angle EDB$
- $\angle CDB$
- $\angle ADC$
- $\angle ADE$

Complete.

- $m\angle 7 + m\angle 6 = m\angle \underline{\quad?}$
- $m\angle 6 + m\angle 5 = m\angle \underline{\quad?}$
- $m\angle 2 + m\angle 3 = \underline{\quad?}$
- If \overrightarrow{DB} bisects $\angle CDA$, then $\angle \underline{\quad?} \cong \angle \underline{\quad?}$.

State the measure of each angle.

- $\angle BOC$
- $\angle GOH$
- $\angle FOG$
- $\angle COF$
- $\angle GOB$
- $\angle HOA$
- Name four angles that are adjacent to $\angle FOG$.
- What ray bisects which two angles?
- Name a pair of congruent:
 - acute angles
 - right angles
 - obtuse angles

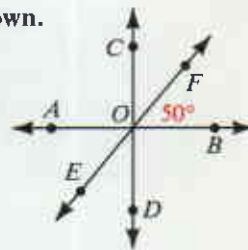


Exs. 17-25

- Study a corner of your classroom where two walls and the ceiling meet. How many right angles can you see at the corner?
- Draw an angle, $\angle AOB$, on a sheet of paper. Fold the paper so that \overrightarrow{OA} falls on \overrightarrow{OB} . Lay the paper flat and call the fold line \overrightarrow{OK} . How is \overrightarrow{OK} related to $\angle AOB$? Explain.

Given the diagram, state whether you can reach the conclusion shown.

- $m\angle FOB = 50$
- $m\angle AOC = 90$
- $m\angle DOC = 180$
- $AO = OB$
- $\angle AOC \cong \angle BOC$
- $m\angle AOF = 130$
- Points E , O , and F are collinear.
- Point C is in the interior of $\angle AOF$.
- $\angle AOE$ and $\angle AOD$ are adjacent angles.
- $\angle AOB$ is a straight angle.
- \overrightarrow{OA} and \overrightarrow{OB} are opposite rays.



Exs. 28-38

Written Exercises

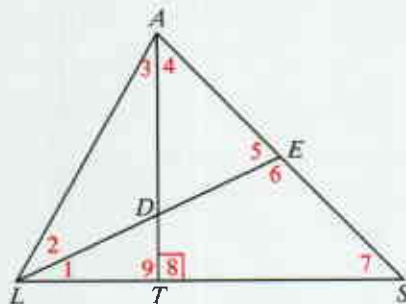
- A**
1. Name the vertex and the sides of $\angle 5$.
 2. Name all angles adjacent to $\angle ADE$.

State another name for the angle.

- | | | |
|-----------------|-----------------|-----------------|
| 3. $\angle 1$ | 4. $\angle 3$ | 5. $\angle 5$ |
| 6. $\angle ALD$ | 7. $\angle AST$ | 8. $\angle LES$ |

State whether the angle appears to be acute, right, obtuse, or straight.

- | | | |
|----------------|------------------|------------------|
| 9. $\angle 2$ | 10. $\angle LAS$ | 11. $\angle ATL$ |
| 12. $\angle S$ | 13. $\angle LTS$ | 14. $\angle EDT$ |



Exs. 1-18

Complete.

- | | |
|---|--|
| 15. $m\angle 3 + m\angle 4 = m\angle \underline{\quad?}$ | 16. $m\angle ALS - m\angle 2 = m\angle \underline{\quad?}$ |
| 17. If $m\angle 1 = m\angle 2$, then $\underline{\quad?}$ bisects $\underline{\quad?}$. | 18. $m\angle LDA + m\angle ADE = \underline{\quad?}$ |

Without measuring, sketch each angle. Then use a protractor to check your accuracy.

- | | | | |
|----------------------|----------------------|-----------------------|----------------------|
| 19. 90° angle | 20. 45° angle | 21. 150° angle | 22. 10° angle |
|----------------------|----------------------|-----------------------|----------------------|

Draw a line, \overleftrightarrow{AB} . Choose a point O between A and B . Use a protractor to investigate the following questions.

23. In the plane represented by your paper, how many lines can you draw through O that will form a 30° angle with \overrightarrow{OB} ?
24. In the plane represented by your paper, how many lines can you draw through O that will form a 90° angle with \overrightarrow{OB} ?

- B**
25. Using a ruler, draw a large triangle. Then use a protractor to find the approximate measure of each angle and compute the sum of the three measures. Repeat this exercise for a triangle with a different shape. Did you get the same result?

26. Find $m\angle 2$, $m\angle 3$, and $m\angle 4$ when the measure of $\angle 1$ is:

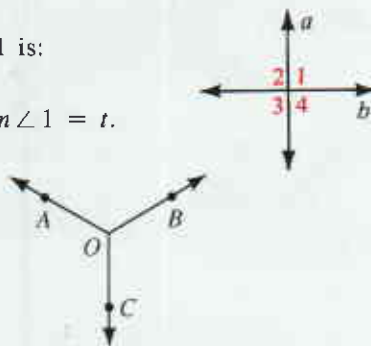
a. 90	b. 93
-------	-------

27. Express $m\angle 2$, $m\angle 3$, and $m\angle 4$ in terms of t when $m\angle 1 = t$.

28. A careless person wrote, using the figure shown,

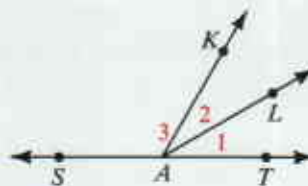
$$m\angle AOB + m\angle BOC = m\angle AOC.$$

What part of the Angle Addition Postulate did that person overlook?



\vec{AL} bisects $\angle KAT$. Find the value of x .

29. $m\angle 3 = 6x$, $m\angle KAT = 90 - x$
 30. $m\angle 1 = 7x + 3$, $m\angle 2 = 6x + 7$
 31. $m\angle 1 = 5x - 12$, $m\angle 2 = 3x + 6$
 32. $m\angle 1 = x$, $m\angle 3 = 4x$
 33. $m\angle 1 = 2x - 8$, $m\angle 3 = 116$
 34. $m\angle 2 = x + 12$, $m\angle 3 = 6x - 20$



Exs. 29-34

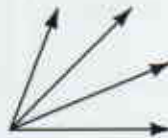
- C 35. a. Complete.



2 rays
1 angle



3 rays
3 angles



4 rays
? angles



5 rays
? angles

- b. Study the pattern in the four cases shown, and predict the number of angles formed by six noncollinear rays that have the same endpoint.
 c. Which of the expressions below gives the number of angles formed by n noncollinear rays that have the same endpoint?

$n - 1$ $2n - 3$ $n^2 - 3$ $\frac{n(n - 1)}{2}$

36. \vec{OC} bisects $\angle AOB$, \vec{OD} bisects $\angle AOC$, \vec{OE} bisects $\angle AOD$, \vec{OF} bisects $\angle AOE$, and \vec{OG} bisects $\angle FOC$.

- a. If $m\angle BOF = 120$, then $m\angle DOE = \underline{\quad? \quad}$.
 b. If $m\angle COG = 35$, then $m\angle EOG = \underline{\quad? \quad}$.

1-5 Postulates and Theorems Relating Points, Lines, and Planes

Recall that we have accepted, without proof, the following four basic assumptions.

The Ruler Postulate

The Segment Addition Postulate

The Protractor Postulate

The Angle Addition Postulate

These postulates deal with segments, lengths, angles, and measures. The following five basic assumptions deal with the way points, lines, and planes are related.

Postulate 5

A line contains at least two points; a plane contains at least three points not all in one line; space contains at least four points not all in one plane.

Postulate 6

Through any two points there is exactly one line.

Postulate 7

Through any three points there is at least one plane, and through any three noncollinear points there is exactly one plane.

Postulate 8

If two points are in a plane, then the line that contains the points is in that plane.

Postulate 9

If two planes intersect, then their intersection is a line.

Important statements that are *proved* are called **theorems**. In Classroom Exercise 1 you will see how Theorem 1-1 follows from the postulates. In Written Exercise 20 you will complete an argument that justifies Theorem 1-2. You will learn about writing proofs in the next chapter.

Theorem 1-1

If two lines intersect, then they intersect in exactly one point.

Theorem 1-2

Through a line and a point not in the line there is exactly one plane.

Theorem 1-3

If two lines intersect, then exactly one plane contains the lines.

The phrase “exactly one” appears several times in the postulates and theorems of this section. The phrase “one and only one” has the same meaning. For example, here is another correct form of Theorem 1-1:

If two lines intersect, then they intersect in one and only one point.

The theorem states that a point of intersection *exists* (there is *at least one* point of intersection) and the point of intersection is *unique* (*no more than one* such point exists).

Classroom Exercises

- Theorem 1-1 states that two lines intersect in exactly one point. The diagram suggests what would happen if you tried to show two “lines” drawn through two points. State the postulate that makes this situation impossible.
- State Postulate 6 using the phrase *one and only one*.
- Reword the following statement as two statements, one describing existence and the other describing uniqueness:
A segment has exactly one midpoint.

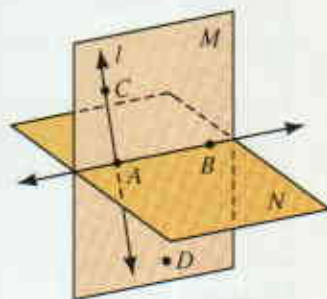


Postulate 6 is sometimes stated as “Two points *determine* a line.”

- Restate Theorem 1-2 using the word *determine*.
- Do two intersecting lines determine a plane?
- Do three points determine a line?
- Do three points determine a plane?

State a postulate, or part of a postulate, that justifies your answer to each exercise.

- Name two points that determine line l .
- Name three points that determine plane M .
- Name the intersection of planes M and N .
- Does \overleftrightarrow{AD} lie in plane M ?
- Does plane N contain any points not on \overleftrightarrow{AB} ?



Surveyors and photographers use a *tripod* for support.

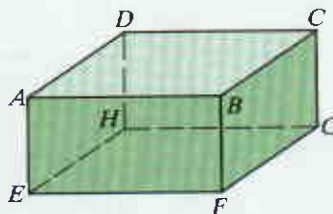
- Why does a three-legged support work better than one with four legs?
- Explain why a four-legged table may rock even if the floor is level.
- A carpenter checks to see if a board is warped by laying a straightedge across the board in several directions. State the postulate that is related to this procedure.
- Think of the intersection of the ceiling and the front wall of your classroom as line l . Let the point in the center of the floor be point C .
 - Is there a plane that contains line l and point C ?
 - State the theorem that applies.



Written Exercises

- A**
1. State Theorem 1-2 using the phrase *one and only one*.
 2. Reword Theorem 1-3 as two statements, one describing existence and the other describing uniqueness.
 3. Planes M and N are known to intersect.
 - a. What kind of figure is the intersection of M and N ?
 - b. State the postulate that supports your answer to part (a).
 4. Points A and B are known to lie in a plane.
 - a. What can you say about \overleftrightarrow{AB} ?
 - b. State the postulate that supports your answer to part (a).

In Exercises 5-11 you will have to visualize certain lines and planes not shown in the diagram of the box. When you name a plane, name it by using four points, no three of which are collinear.



Exs. 5-12

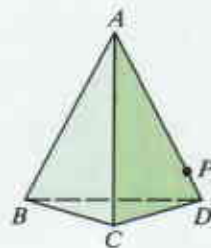
5. Write the postulate that assures you that \overleftrightarrow{AC} exists.
6. Name a plane that contains \overleftrightarrow{AC} .
7. Name a plane that contains \overleftrightarrow{AC} but that is not shown in the diagram.
8. Name the intersection of plane $DCFE$ and plane $ABCD$.
9. Name four lines shown in the diagram that don't intersect plane $EFGH$.
10. Name two lines that are not shown in the diagram and that don't intersect plane $EFGH$.
11. Name three planes that don't intersect \overleftrightarrow{EF} and don't contain \overleftrightarrow{EF} .
12. If you measure $\angle EFG$ with a protractor you get more than 90° . But you know that $\angle EFG$ represents a right angle in a box. Using this as an example, complete the table.

	$\angle EFG$	$\angle AEF$	$\angle DCB$	$\angle FBC$
In the diagram	obtuse	?	?	?
In the box	right	?	?	?

State whether it is possible for the figure described to exist. Write *yes* or *no*.

- B**
13. Two points both lie in each of two lines.
 14. Three points all lie in each of two planes.
 15. Three noncollinear points all lie in each of two planes.
 16. Two points lie in a plane X , two other points lie in a different plane Y , and the four points are coplanar but not collinear.

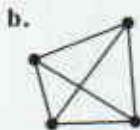
17. Points R , S , and T are noncollinear points.
- State the postulate that guarantees the existence of a plane X that contains R , S , and T .
 - Draw a diagram showing plane X containing the noncollinear points R , S , and T .
 - Suppose that P is any point of \overleftrightarrow{RS} other than R and S . Does point P lie in plane X ? Explain.
 - State the postulate that guarantees that \overleftrightarrow{TP} exists.
 - State the postulate that guarantees that \overleftrightarrow{TP} is in Plane X .
18. Points A , B , C , and D are four noncoplanar points.
- State the postulate that guarantees the existence of planes ABC , ABD , ACD , and BCD .
 - Explain how the Ruler Postulate guarantees the existence of a point P between A and D .
 - State the postulate that guarantees the existence of plane BCP .
 - Explain why there are an infinite number of planes through \overline{BC} .



- C** 19. State how many segments can be drawn between the points in each figure. No three points are collinear.



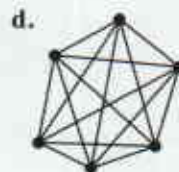
3 points
? segments



4 points
? segments

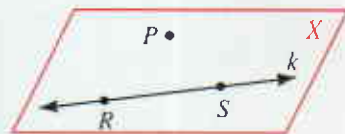
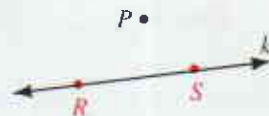


5 points
? segments



6 points
? segments

- Without making a drawing, predict how many segments can be drawn between seven points, no three of which are collinear.
 - How many segments can be drawn between n points, no three of which are collinear?
20. Parts (a) through (d) justify Theorem 1-2: Through a line and a point not in the line there is exactly one plane.
- If P is a point not in line k , what postulate permits us to state that there are two points R and S in line k ?
 - Then there is at least one plane X that contains points P , R , and S . Why?
 - What postulate guarantees that plane X contains line k ? Now we know that there is a plane X that contains both point P and line k .
 - There can't be another plane that contains point P and line k , because then *two* planes would contain noncollinear points P , R , and S . What postulate does this contradict?



Application

Locating Points

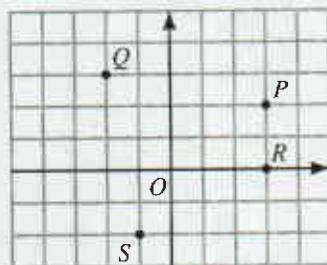
Suppose you lived in an area with streets laid out on a grid. If you lived in a house located at point P in the diagram at the right below, you could tell someone where you lived by saying:

From the crossing at the center of town,
go three blocks east and two blocks north.

A friend of yours living at Q might say she lives two blocks west and three blocks north of the town center.

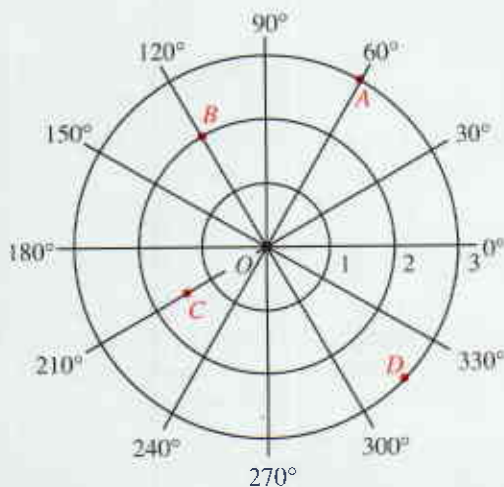
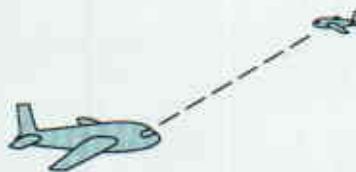


Mathematicians make such descriptions shorter by using a grid system and *coordinates*. They use $(3, 2)$ for your house at point P , and $(-2, 3)$ for your friend's house at Q . Point O at the center of town is $(0, 0)$. Points R and S are $(3, 0)$ and $(-1, -2)$.



This grid system is not always the easiest way to describe a position. If you were a pilot and saw another airplane while flying, it would be difficult to give its position in this system. However, you might say the other plane is 4 km away at 11 o'clock, with 12 o'clock being straight ahead.

Mathematicians sometimes find it convenient to describe a point by a distance and an angle. Rotation in a clockwise direction is represented by a negative angle. Counterclockwise rotation is represented by a positive angle. A complete rotation, all the way around once, is 360° (or -360°). The labeled points in the diagram at the right are described as shown below.



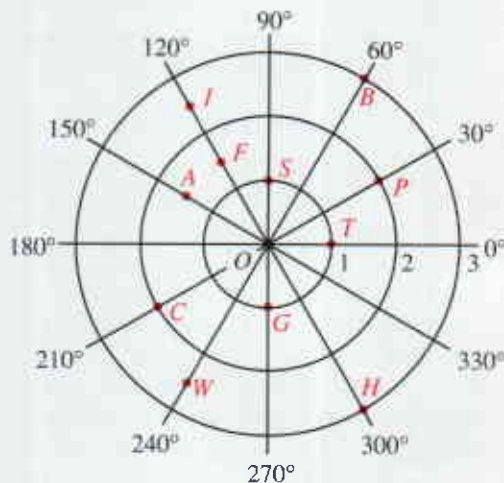
A	$(3, 60^\circ)$
B	$(2, 120^\circ)$
C	$(1.5, 210^\circ)$ or $(1.5, -150^\circ)$
D	$(3, 315^\circ)$ or $(3, -45^\circ)$

Sometimes you may want to change from one system to the other. For example, if you were at the town center and walked two blocks east and three blocks north, what would your position be in the distance-angle system? Use a centimeter ruler and draw the triangle suggested by your path. If you measure the triangle, you will get about $(3.6, 56^\circ)$.



Exercises

- Copy the grid system shown on the previous page onto a piece of graph paper. Then locate the following points.
 - A point T five blocks due west of the center of town
 - A point U five blocks east and two blocks south of the center of town
 - A point V two blocks west and one block north of your house, which is located at point P



- Give the letter that names each point.
 - $(2, 30^\circ)$
 - $(2.5, 120^\circ)$
 - $(1, -90^\circ)$
- Give the distance and angle for each point.
 - C
 - A
 - T
- Give another way of naming each point.
 - $(1, -120^\circ)$
 - $(2, 300^\circ)$
 - $(2.5, -180^\circ)$
- A point is given in the grid system. What would it be called in the distance-angle system? (*Hint:* See the discussion at the top of the page. Use a protractor and a centimeter ruler to help you answer the question.)
 - $(3, 4)$
 - $(-2, 5)$
 - $(4, 0)$
 - $(8, -6)$
- A point is given in the distance-angle system. What would it be called, approximately, in the grid system? (*Hint:* Use a protractor and a centimeter ruler to draw the triangle suggested by the angle and distance. Measure the sides of the triangle.)
 - $(2, 50^\circ)$
 - $(1.5, -70^\circ)$
 - $(3, 90^\circ)$
 - $(1, 120^\circ)$

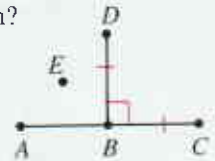
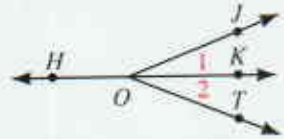
Self-Test 2

- Write three names for the line pictured.
- Name the ray that is opposite to \overrightarrow{NC} .
- Is it correct to say that point B lies between points N and C ?
- When $RN = 7$, $NC = 3x + 5$, and $RC = 18$, what is the value of x ?



Complete.

- $m\angle 1 + m\angle 2 = m\angle \underline{\quad?}$
- If $\angle 1 \cong \angle 2$, then $\underline{\quad?}$ is the bisector of $\angle \underline{\quad?}$.
- $m\angle HOK = \underline{\quad?}$, and $\angle HOK$ is called a(n) $\underline{\quad?}$ angle.
- Which of the four things stated *can't* you conclude from the diagram?
 - A , B , and C are collinear.
 - $\angle DBC$ is a right angle.
 - B is the midpoint of \overline{AC} .
 - E is in the interior of $\angle DBA$.



Apply postulates and theorems to complete the statements.

- Through any two points $\underline{\quad?}$.
- If points A and B are in plane Z , $\underline{\quad?}$.
- If two planes intersect, then $\underline{\quad?}$.
- If there is a line j and a point P not in the line, then $\underline{\quad?}$.

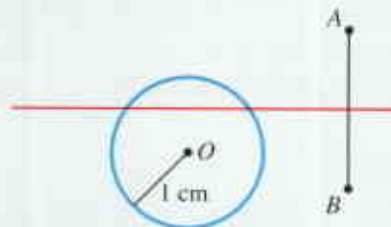
Chapter Summary

- The concepts of *point*, *line*, and *plane* are basic to geometry. These undefined terms are used in the definitions of other terms.
- \overleftrightarrow{AB} represents a line, \overline{AB} a segment, and \overrightarrow{AB} a ray. AB represents the length of \overline{AB} ; AB is a positive number.
- Two rays with the same endpoint form an angle.
- Congruent segments have equal lengths. Congruent angles have equal measures.
- Angles are classified as acute, right, obtuse, or straight, according to their measures.
- Diagrams enable you to reach certain conclusions. However, judgments about segment length and angle measure must not be made on the basis of appearances alone.
- Statements that are accepted without proof are called postulates. Statements that are proved are called theorems.
- Postulates and theorems in this chapter deal with distances, angle measures, points, lines, and planes.

Chapter Review

In Exercises 1–4 answer on the basis of what appears to be true.

- How many blue points are 1 cm from point O ?
- How many red points are 1 cm from O ?
- How many red points are 2 cm from O ?
- Each red point is said to be $\underline{\quad ? \quad}$ from points A and B .



1-1

Sketch and label the figures described.

- Points A , B , C , and D are coplanar, but A , B , and C are the only three of those points that are collinear.
- Line l intersects plane X in point P .
- Plane M contains intersecting lines j and k .
- Planes X and Y intersect in \overleftrightarrow{AB} .

1-2

- Name a point on \overleftrightarrow{ST} that is not on \overleftrightarrow{RT} .

- Complete: $RS = \underline{\quad ? \quad}$ and $ST = \underline{\quad ? \quad}$

- Complete: \overline{RS} and \overline{ST} are called $\underline{\quad ? \quad}$ segments.

- If U is the midpoint of \overline{TV} , find the value of x .



1-3

- Name three angles that have vertex D . Which angles with vertex D are adjacent angles?

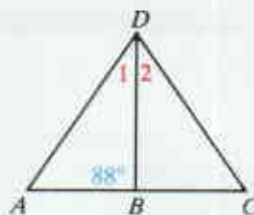
1-4

- $m\angle CBD = \underline{\quad ? \quad}$

- Name the postulate that justifies your answer in part (a).

- What kind of angle is $\angle CBD$?

- \overleftrightarrow{DB} bisects $\angle ADC$, $m\angle 1 = 5x - 3$, and $m\angle 2 = x + 25$. Find the value of x .



Classify each statement as true or false.

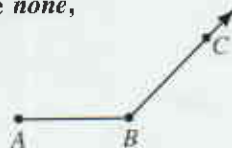
- It is possible to locate three points in such a position that an unlimited number of planes contain all three points.
- It is possible for two intersecting lines to be noncoplanar.
- Through any three points there is at least one line.
- If points A and B lie in plane P , then so does any point of \overleftrightarrow{AB} .

1-5

Chapter Test

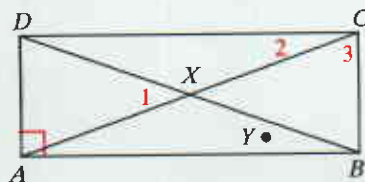
State how many points meet the requirements. For each answer write *none*, *one*, or *an unlimited number*.

- Equidistant from points A and B
- On \overrightarrow{BC} and equidistant from points A and B



Given the diagram, tell whether you can reach the conclusion shown.

- $\angle AXC$ is a straight angle.
- Point Y lies in the interior of $\angle 3$.
- $\angle ADC$ is a right angle.
- X is the midpoint of \overline{AC} .
- Point Y lies between points A and B .
- Name three collinear points.
- Name the intersection of \overrightarrow{CX} and \overleftrightarrow{AB} .
- Which postulate justifies the statement $AX + XC = AC$?
- If \overline{AC} bisects \overline{BD} , name two congruent segments.
- Name the vertex and sides of $\angle 1$.
- Name a right angle.
- If $m\angle 1 = 46$, find $m\angle DXC$ and $m\angle CXB$.
- If $m\angle DAX = 70$, find the measure of $\angle XAB$.



Exs. 3-15

Exercises 16–20 refer to a number line that is not pictured here. Point A has coordinate 2 and point B has coordinate 5.

- What is the length of \overline{AB} ?
- What is the coordinate of the midpoint of \overline{AB} ?
- If A is the midpoint of \overline{PB} , what is the coordinate of P ?
- What is the coordinate of a point that is on \overrightarrow{AB} and is 4 units from B ?
- What is the coordinate of a point that is 4 units from B , but is not on \overrightarrow{AB} ?
- Is it possible for a line and a point to be noncoplanar?
- Is it possible for the intersection of two planes to consist of a segment?
- Is a postulate an important proved statement, or is it a basic assumption?
- Complete the statement of the postulate: If two points are in a plane, then .