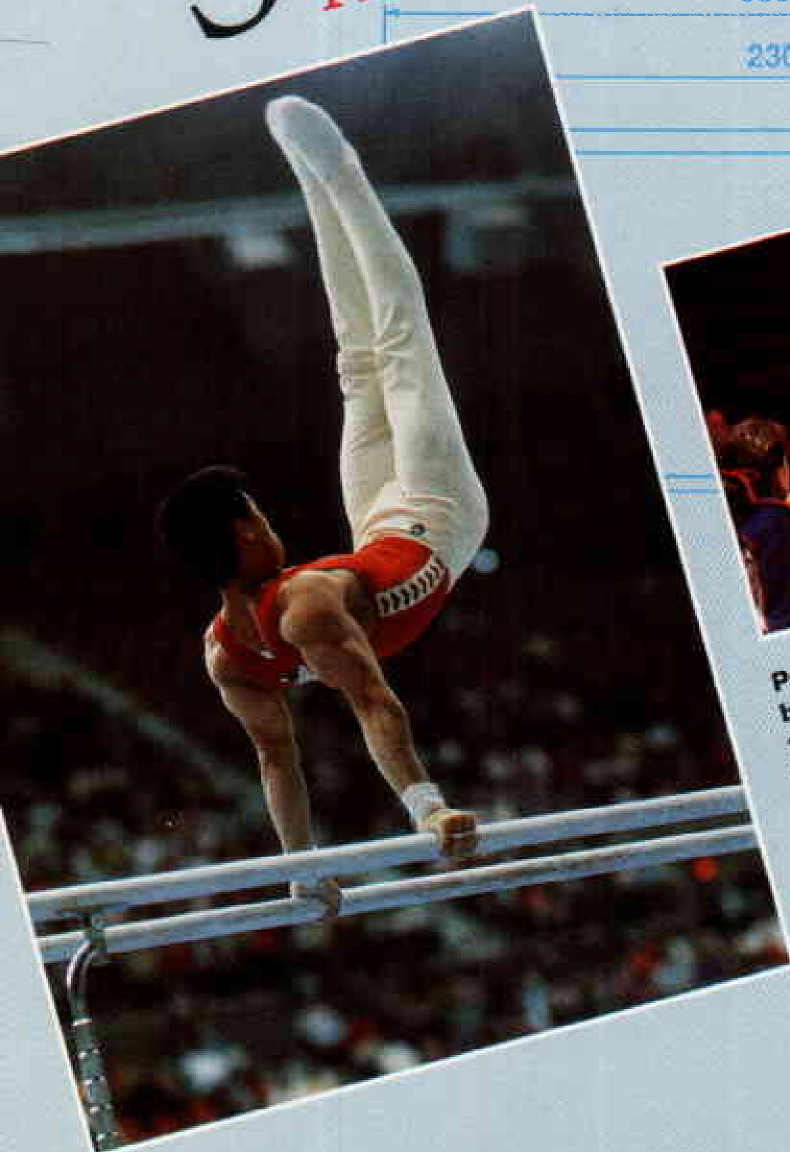
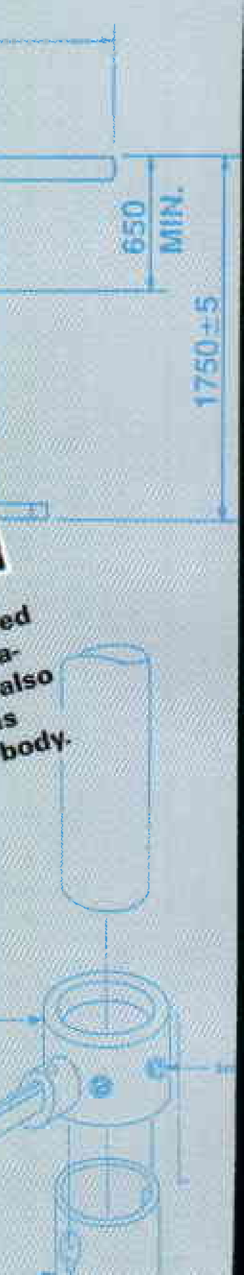
3 PARALLEL LINES AND PLANES





Parallel lines are suggested by many pieces of apparatus in a gymnasium and also by some of the positions taken by a performer's body.

When Lines and Planes Are Parallel

Objectives

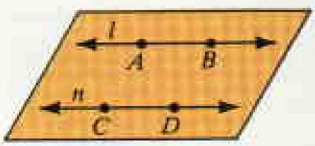
1. Distinguish between intersecting lines, parallel lines, and skew lines.
2. State and apply the theorem about the intersection of two parallel planes by a third plane.
3. Identify the angles formed when two lines are cut by a transversal.
4. State and apply the postulates and theorems about parallel lines.
5. State and apply the theorems about a parallel and a perpendicular to a given line through a point outside the line.

3-1 *Definitions*

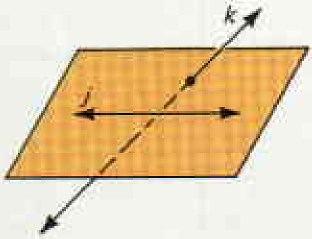
Two lines that do not intersect are either *parallel* or *skew*.

Parallel lines (∥ lines) are coplanar lines that do not intersect.

Skew lines are noncoplanar lines. Therefore, they are neither parallel nor intersecting.



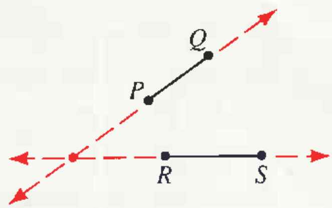
l and n are parallel lines.
l is parallel to n ($l \parallel n$).



j and k are skew lines.

Segments and rays contained in parallel lines are also called parallel. For example, in the figure at the left above, $\overline{AB} \parallel \overline{CD}$ and $\overrightarrow{AB} \parallel \overrightarrow{CD}$.

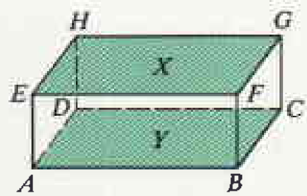
In the diagram at the right, $\overline{PQ}$ and $\overline{RS}$ do not intersect, but they are parts of lines, $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$, that do intersect. Thus $\overline{PQ}$ is *not* parallel to $\overline{RS}$.



The box pictured below may help you understand the following definitions. Think of the top of the box as part of plane X and the bottom of the box as part of plane Y.

Parallel planes (∥ planes) do not intersect.
Plane X is parallel to plane Y ($X \parallel Y$).

A line and a plane are parallel if they do not intersect.
For example, $\overleftrightarrow{EF} \parallel Y$ and $\overleftrightarrow{FG} \parallel Y$.
Also, $\overleftrightarrow{AB} \parallel X$ and $\overleftrightarrow{BC} \parallel X$.



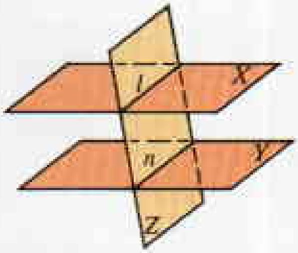
Our first theorem about parallel lines and planes is given below. Notice the importance of definitions in the proof.

Theorem 3-1

If two parallel planes are cut by a third plane, then the lines of intersection are parallel.

Given: Plane $X \parallel$ plane Y;
plane Z intersects X in line l;
plane Z intersects Y in line n.

Prove: $l \parallel n$

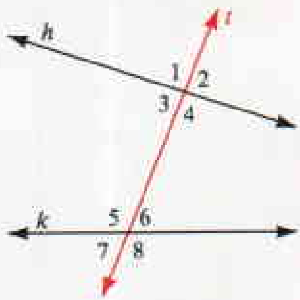


Proof:

Statements	Reasons
1. l is in Z; n is in Z.	1. Given
2. l and n are coplanar.	2. Def. of coplanar
3. l is in X; n is in Y; $X \parallel Y$.	3. Given
4. l and n do not intersect.	4. Parallel planes do not intersect. (Def. of $\parallel$ planes)
5. $l \parallel n$	5. Def. of $\parallel$ lines (Steps 2 and 4)

The following terms, which are needed for future theorems about parallel lines, apply only to coplanar lines.

A **transversal** is a line that intersects two or more coplanar lines in different points. In the next diagram, t is a transversal of h and k. The angles formed have special names.



Interior angles: angles 3, 4, 5, 6

Exterior angles: angles 1, 2, 7, 8

Alternate interior angles (alt. int. ∠s) are two nonadjacent interior angles on opposite sides of the transversal.

$\angle 3$ and $\angle 6$ $\qquad$ $\angle 4$ and $\angle 5$

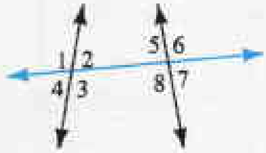
Same-side interior angles (s-s. int. ∠s) are two interior angles on the same side of the transversal.

$\angle 3$ and $\angle 5$ $\qquad$ $\angle 4$ and $\angle 6$

Corresponding angles (corr. ∠s) are two angles in corresponding positions relative to the two lines.

$\angle 1$ and $\angle 5$ $\qquad$ $\angle 2$ and $\angle 6$ $\qquad$ $\angle 3$ and $\angle 7$ $\qquad$ $\angle 4$ and $\angle 8$

Classroom Exercises



1. The blue line is a transversal.

a. Name four pairs of corresponding angles.

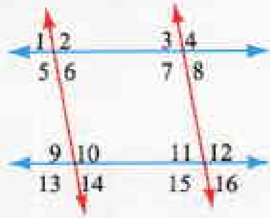
b. Name two pairs of alternate interior angles.

c. Name two pairs of same-side interior angles.

d. Name two pairs of angles that could be called *alternate exterior angles*.

e. Name two pairs of angles that could be called *same-side exterior angles*.

Classify each pair of angles as alternate interior angles, same-side interior angles, corresponding angles, or none of these.



2. $\angle 7$ and $\angle 11$ **3.** $\angle 14$ and $\angle 16$

4. $\angle 4$ and $\angle 10$ **5.** $\angle 3$ and $\angle 6$

6. $\angle 6$ and $\angle 11$ **7.** $\angle 2$ and $\angle 10$

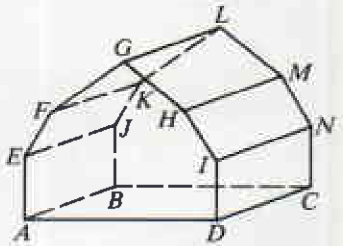
8. $\angle 2$ and $\angle 3$ **9.** $\angle 7$ and $\angle 12$

10. Classify each pair of lines as intersecting, parallel, or skew.

a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EJ}$ **b.** $\overleftrightarrow{AB}$ and $\overleftrightarrow{FK}$

c. $\overleftrightarrow{AB}$ and $\overleftrightarrow{ID}$ **d.** $\overleftrightarrow{EF}$ and $\overleftrightarrow{IH}$

e. $\overleftrightarrow{EF}$ and $\overleftrightarrow{NM}$ **f.** $\overleftrightarrow{CN}$ and $\overleftrightarrow{FG}$



11. Name six lines parallel to $\overleftrightarrow{GL}$.

12. Name several lines skew to $\overleftrightarrow{GL}$.

13. Name five lines parallel to plane *ABCD*.

14. Name two coplanar segments that do not intersect and yet are not parallel.

Complete each statement with the word *always*, *sometimes*, or *never*.

15. Two skew lines are __?__ parallel.

16. Two parallel lines are __?__ coplanar.

17. A line in the plane of the ceiling and a line in the plane of the floor are __?__ parallel.

18. Two lines in the plane of the floor are __?__ skew.

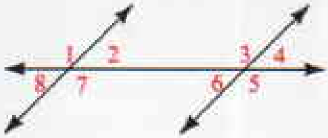
19. A line in the plane of a wall and a line in the plane of the floor are

a. __?__ parallel. **b.** __?__ intersecting. **c.** __?__ skew.

Written Exercises

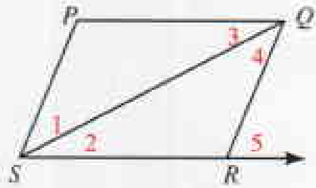
Classify each pair of angles as alternate interior angles, same-side interior angles, or corresponding angles.

A 1. $\angle 2$ and $\angle 6$
2. $\angle 8$ and $\angle 6$
3. $\angle 2$ and $\angle 3$
4. $\angle 3$ and $\angle 7$
5. $\angle 5$ and $\angle 7$
6. $\angle 3$ and $\angle 1$



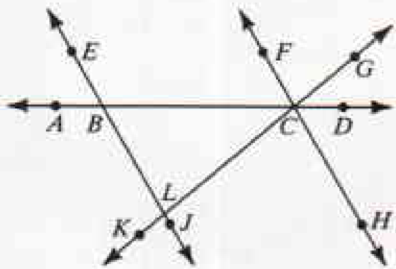
Name the two lines and the transversal that form each pair of angles.

7. $\angle 2$ and $\angle 3$
8. $\angle 1$ and $\angle 4$
9. $\angle P$ and $\angle PSR$
10. $\angle 5$ and $\angle PSR$
11. $\angle 5$ and $\angle PQR$



Classify each pair of angles as alternate interior, same-side interior, or corresponding angles.

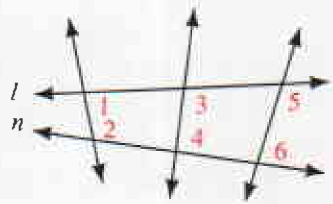
12. $\angle EBA$ and $\angle FCB$
13. $\angle DCH$ and $\angle CBJ$
14. $\angle FCB$ and $\angle CBL$
15. $\angle FCL$ and $\angle BLC$
16. $\angle HCB$ and $\angle CBJ$
17. $\angle GCH$ and $\angle GLJ$



In Exercises 18–20 use two lines of notebook paper as parallel lines and draw any transversal. Use a protractor to measure.

18. Measure one pair of corresponding angles. Repeat the experiment with another transversal. What appears to be true?
19. Measure one pair of alternate interior angles. Repeat the experiment with another transversal. What appears to be true?
20. Measure one pair of same-side interior angles. Repeat the experiment with another transversal. What appears to be true?

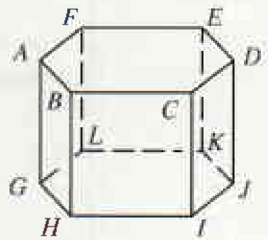
B 21. Draw a large diagram showing three transversals intersecting two nonparallel lines l and n. Number three pairs of same-side interior angles *on the same sides of the transversals*, as shown in the diagram.
   a. Find $m\angle 1 + m\angle 2$.
   b. Find $m\angle 3 + m\angle 4$.
   c. Predict the value of $m\angle 5 + m\angle 6$. Then check your prediction by measuring.
   d. What do you conclude?



22. Draw a diagram of a six-sided box by following the steps below.

Step 1

Draw a six-sided top. Then draw an exact copy of the top directly below it.



Step 2

Draw vertical edges. Make invisible edges dashed.

Exercises 23–29 refer to the diagram in Step 2 of Exercise 22.

23. Name five lines that appear to be parallel to $\overline{AG}$.

24. Name three lines that appear to be parallel to $\overline{AB}$.

25. Name four lines that appear to be skew to $\overline{AB}$.

26. Name two planes parallel to $\overleftrightarrow{AF}$.

27. Name four planes parallel to $\overleftrightarrow{FL}$.

28. How many pairs of parallel planes are shown?

29. Suppose the top and bottom of the box lie in parallel planes. Explain how Theorem 3-1 can be used to prove $\overline{CD} \parallel \overline{IJ}$.

Complete each statement with the word *always*, *sometimes*, or *never*.

30. When there is a transversal of two lines, the three lines are _?_ coplanar.

31. Three lines intersecting in one point are _?_ coplanar.

32. Two lines that are not coplanar _?_ intersect.

33. Two lines parallel to a third line are _?_ parallel to each other.

34. Two lines skew to a third line are _?_ skew to each other.

35. Two lines perpendicular to a third line are _?_ perpendicular to each other.

36. Two planes parallel to the same line are _?_ parallel to each other.

37. Two planes parallel to the same plane are _?_ parallel to each other.

38. Lines in two parallel planes are _?_ parallel to each other.

39. Two lines parallel to the same plane are _?_ parallel to each other.

Draw each figure described.

C **40.** Lines a and b are skew, lines b and c are skew, and $a \parallel c$.

41. Lines d and e are skew, lines e and f are skew, and $d \perp f$.

42. Line $l \parallel$ plane X, plane $X \parallel$ plane Y, and l is not parallel to Y.

Explorations

These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.

Draw two parallel segments and a transversal and label the points of intersection. Measure all eight angles formed. Repeat several times. Do you notice any patterns? What kinds of angles are congruent? What kinds of angles are supplementary?

3-2 *Properties of Parallel Lines*

By experimenting with parallel lines, transversals, and a protractor in Exercise 18, page 76, you probably discovered that corresponding angles are congruent. There is not enough information in our previous postulates and theorems to deduce this property as a theorem. We will accept it as a postulate.

Postulate 10

If two parallel lines are cut by a transversal, then corresponding angles are congruent.

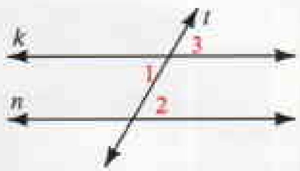
From this postulate we can easily prove the next three theorems.

Theorem 3-2

If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

Given: $k \parallel n$; transversal t cuts k and n.

Prove: $\angle 1 \cong \angle 2$



Proof:

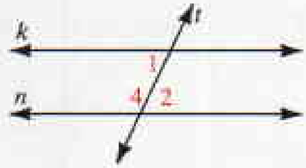
Statements	Reasons
1. $k \parallel n$	1. Given
2. $\angle 1 \cong \angle 3$	2. Vert. $\angle$s are $\cong$.
3. $\angle 3 \cong \angle 2$	3. If two parallel lines are cut by a transversal, then corr. $\angle$s are $\cong$.
4. $\angle 1 \cong \angle 2$	4. Transitive Property

Theorem 3-3

If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.

Given: $k \parallel n$; transversal t cuts k and n.

Prove: $\angle 1$ is supplementary to $\angle 4$.



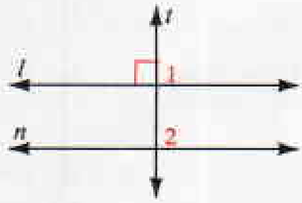
The proof is left as Exercise 22.

Theorem 3-4

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other one also.

Given: Transversal t cuts l and n;
$t \perp l$; $l \parallel n$

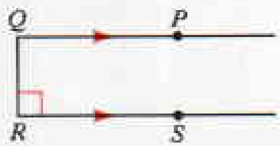
Prove: $t \perp n$



The proof is left as Exercise 13.

For the rest of this book, arrowheads will no longer be used in diagrams to suggest that a line extends in both directions without ending. Instead, pairs of arrowheads (and double arrowheads when necessary) will be used to indicate parallel lines, as shown in the following examples.

Example 1 Find the measure of $\angle PQR$.

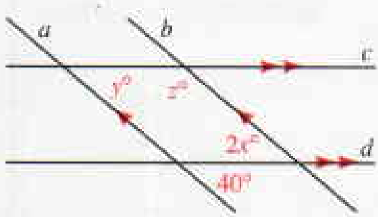


Solution The diagram shows that

$$\overleftrightarrow{QR} \perp \overleftrightarrow{RS} \text{ and } \overleftrightarrow{QP} \parallel \overleftrightarrow{RS}.$$

Then by Theorem 3-4, $\overleftrightarrow{QR} \perp \overleftrightarrow{QP}$ and $m\angle PQR = 90$.

Example 2 Find the values of x, y, and z.

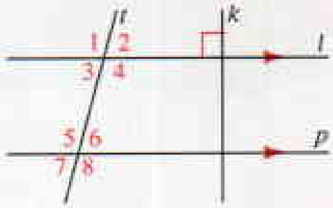


Solution

Since $a \parallel b$, $2x = 40$. (Why?)
Thus, $x = 20$.
Since $c \parallel d$, $y = 40$. (Why?)
Since $a \parallel b$, $y + z = 180$. (Why?)
$40 + z = 180$
$z = 140$

Classroom Exercises

1. What do the arrowheads in the diagram tell you?



Exs. 1–13

State the postulate or theorem that justifies each statement.

2. $\angle 1 \cong \angle 5$ **3.** $\angle 3 \cong \angle 6$

4. $m\angle 4 + m\angle 6 = 180$ **5.** $m\angle 4 = m\angle 8$

6. $m\angle 4 = m\angle 5$ **7.** $\angle 6 \cong \angle 7$

8. $k \perp p$ **9.** $\angle 3$ is supplementary to $\angle 5$.

10. If $m\angle 1 = 130$, what are the measures of the other numbered angles?

11. If $m\angle 1 = x$, what are the measures of the other numbered angles?

12. If $m\angle 4 = 2m\angle 3$, find $m\angle 6$.

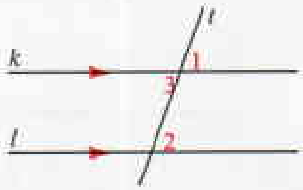
13. If $m\angle 5 = m\angle 6 + 20$, find $m\angle 1$.

14. Alan tried to prove Postulate 10 as shown below. However, he did *not* have a valid proof. Explain why not.

If two parallel lines are cut by a transversal, then corresponding angles are congruent.

Given: $k \parallel l$; transversal t cuts k and l.

Prove: $\angle 1 \cong \angle 2$



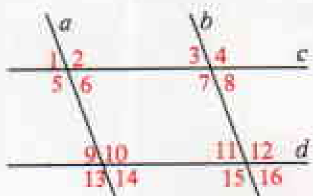
Proof:

Statements	Reasons
1. $k \parallel l$	1. Given
2. $\angle 3 \cong \angle 2$	2. If two parallel lines are cut by a transversal, then alt. int. ∠s are ≅.
3. $\angle 1 \cong \angle 3$	3. Vert. ∠s are ≅.
4. $\angle 1 \cong \angle 2$	4. Transitive Prop.

Written Exercises

A **1.** If $a \parallel b$, name all angles that must be congruent to $\angle 1$.

2. If $c \parallel d$, name all angles that must be congruent to $\angle 1$.



Exs. 1–6

Assume that $a \parallel b$ and $c \parallel d$.

3. Name all angles congruent to $\angle 2$.

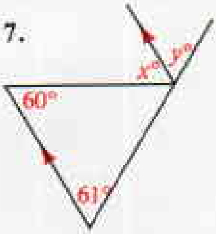
4. Name all angles supplementary to $\angle 2$.

5. If $m\angle 13 = 110$, then $m\angle 15 =$ __?__ and $m\angle 3 =$ __?__.

6. If $m\angle 7 = x$, then $m\angle 12 =$ __?__ and $m\angle 6 =$ __?__.

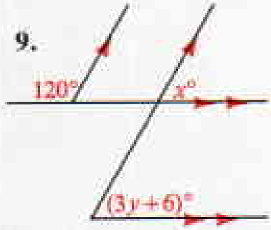
Find the values of x and y.

7.



8.

9.



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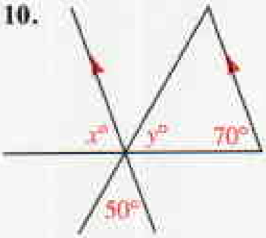

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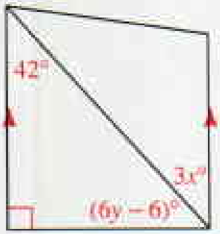

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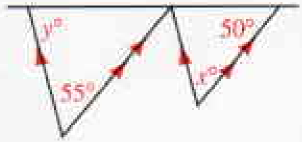
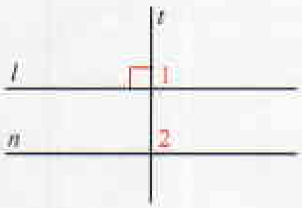


13. Copy and complete the proof of Theorem 3-4.

Given: Transversal t cuts l and n;
$t \perp l$; $l \parallel n$

Prove: $t \perp n$

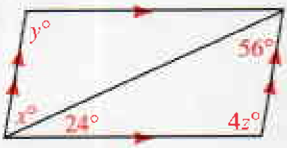


Proof:

Statements	Reasons
1. $t \perp l$	1. ___?___
2. $m\angle 1 = 90$	2. ___?___
3. ___?___	3. Given
4. $\angle 2 \cong \angle 1$ or $m\angle 2 = m\angle 1$	4. ___?___
5. ___?___	5. Substitution Property
6. $t \perp n$	6. ___?___

Find the values of x, y, and z.

B **14.**



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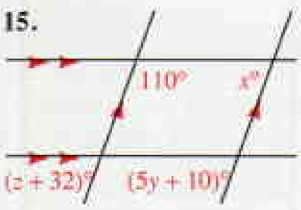

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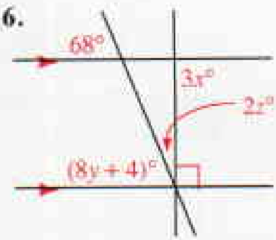
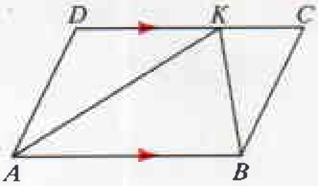


17. Given: $\overline{AB} \parallel \overline{CD}$; $m\angle D = 116$;
$\overrightarrow{AK}$ bisects $\angle DAB$.

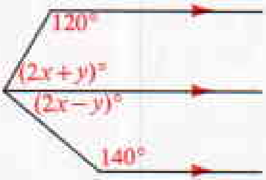
a. Find the measures of $\angle DAB$, $\angle KAB$, and $\angle DKA$.

b. Is there enough information for you to conclude that $\angle D$ and $\angle C$ are supplementary, or is more information needed?



Find the values of *x* and *y*.

18.



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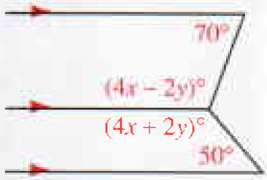
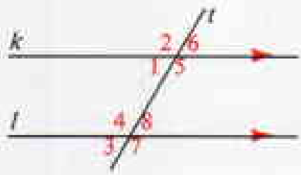


Write proofs in two-column form.

20. Given: $k \parallel l$
Prove: $\angle 2 \cong \angle 7$

21. Given: $k \parallel l$
Prove: $\angle 1$ is supplementary to $\angle 7$.



22. Copy what is shown for Theorem 3-3 on page 79. Then write a proof in two-column form.

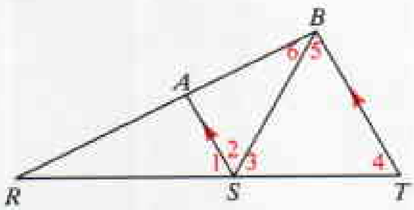
23. Draw a four-sided figure *ABCD* with $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$.

a. Prove that $\angle A \cong \angle C$.

b. Is $\angle B \cong \angle D$?

C **24.** Given: $\overline{AS} \parallel \overline{BT}$;
$m\angle 4 = m\angle 5$
Prove: $\overrightarrow{SA}$ bisects $\angle BSR$.

25. Given: $\overline{AS} \parallel \overline{BT}$;
$m\angle 4 = m\angle 5$;
$\overrightarrow{SB}$ bisects $\angle AST$.
Find the measure of $\angle 1$.



Mixed Review Exercises

For each statement (a) tell whether the statement is true or false, (b) write the converse, and (c) tell whether the converse is true or false.

1. If two lines are perpendicular, then they form congruent adjacent angles.
2. If two lines are parallel, then they are not skew.
3. Two angles are supplementary if the sum of their measures is 180.
4. Two planes are parallel only if they do not intersect.

3-3 *Proving Lines Parallel*

In the preceding section you saw that when two lines are parallel, you can conclude that certain angles are congruent or supplementary. In this section the situation is reversed. From two angles being congruent or supplementary you will conclude that certain lines forming the angles are parallel. The key to doing this is Postulate 11 below. Postulate 10 is repeated so you can compare the wording of the postulates. Notice that these two postulates are converses of each other.

Postulate 10

If two parallel lines are cut by a transversal, then corresponding angles are congruent.

Postulate 11

If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel.

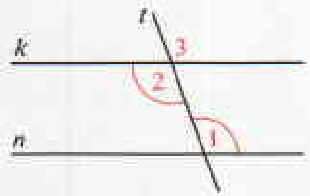
The next three theorems can be deduced from Postulate 11.

Theorem 3-5

If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

Given: Transversal t cuts lines k and n;
$\angle 1 \cong \angle 2$

Prove: $k \parallel n$



Proof:

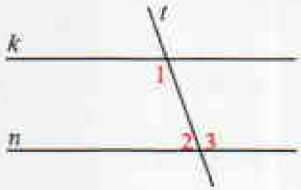
Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 2 \cong \angle 3$	2. Vert. ∠s are ≅.
3. $\angle 1 \cong \angle 3$	3. Transitive Property
4. $k \parallel n$	4. If two lines are cut by a transversal and corr. ∠s are ≅, then the lines are ∥.

You may have recognized that Theorem 3-5 is the converse of Theorem 3-2, "If two parallel lines are cut by a transversal, then alternate interior angles are congruent." The next theorem is the converse of Theorem 3-3, "If two parallel lines are cut by a transversal, then same-side interior angles are supplementary."

Theorem 3-6

If two lines are cut by a transversal and same-side interior angles are supplementary, then the lines are parallel.

Given: Transversal t cuts lines k and n; $\angle 1$ is supplementary to $\angle 2$.
Prove: $k \parallel n$

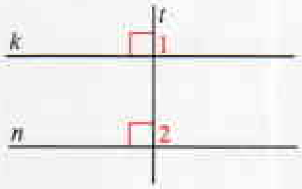


The proof is left as Exercise 22.

Theorem 3-7

In a plane two lines perpendicular to the same line are parallel.

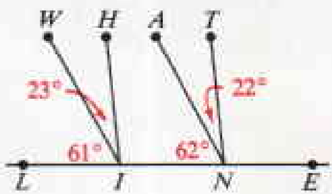
Given: $k \perp t$; $n \perp t$
Prove: $k \parallel n$



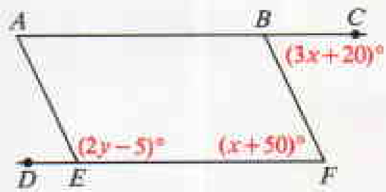
The proof is left as Exercise 23.

Example 1 Which segments are parallel?

Solution (1) $\overline{HI}$ and $\overline{TN}$ are parallel since corresponding angles have the same measure:
$m\angle HIL = 23 + 61 = 84$
$m\angle TNI = 22 + 62 = 84$
(2) $\overline{WI}$ and $\overline{AN}$ are *not* parallel since $61 \neq 62$.



Example 2 Find the values of x and y that make $\overline{AC} \parallel \overline{DF}$ and $\overline{AE} \parallel \overline{BF}$.



Solution If $m\angle CBF = m\angle BFE$, then $\overline{AC} \parallel \overline{DF}$. (Why?)

$$3x + 20 = x + 50$$
$$2x = 30$$
$$x = 15$$

If $\angle AEF$ and $\angle F$ are supplementary, then $\overline{AE} \parallel \overline{BF}$. (Why?)

$$(2y - 5) + (x + 50) = 180$$
$$(2y - 5) + (15 + 50) = 180$$
$$2y = 120$$
$$y = 60$$

The following theorems can be proved using previous postulates and theorems. We state the theorems without proof, however, for you to use in future work.

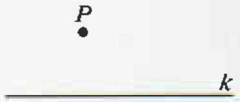
Theorem 3-8

Through a point outside a line, there is exactly one line parallel to the given line.

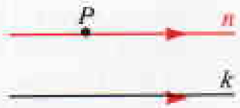
Theorem 3-9

Through a point outside a line, there is exactly one line perpendicular to the given line.

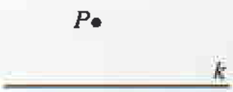
Given this:



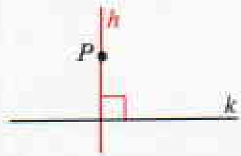
Theorem 3-8 says that line n exists and is unique.



Given this:



Theorem 3-9 says that line h exists and is unique.



Theorem 3-10

Two lines parallel to a third line are parallel to each other.

Given: $k \parallel l$; $k \parallel n$

Prove: $l \parallel n$

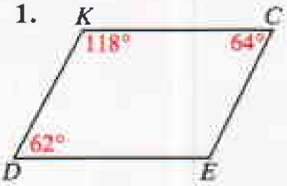
In Classroom Exercise 20 you will explain why this theorem is true when all three lines are coplanar. The theorem also holds true for lines in space.

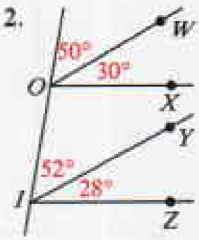
Ways to Prove Two Lines Parallel

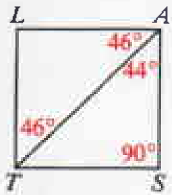
1. Show that a pair of corresponding angles are congruent.
2. Show that a pair of alternate interior angles are congruent.
3. Show that a pair of same-side interior angles are supplementary.
4. In a plane show that both lines are perpendicular to a third line.
5. Show that both lines are parallel to a third line.

Classroom Exercises

State which segments (if any) are parallel. State the postulate or theorem that justifies your answer.

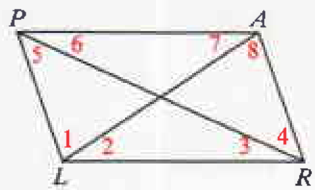
1. 

2. 

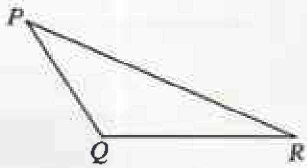
3. 

4. 

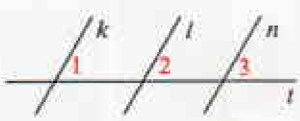
In each exercise some information is given. Use this information to name the segments that must be parallel. If there are no such segments, say so.



5. $m\angle 1 = m\angle 8$
6. $\angle 2 \cong \angle 7$
7. $\angle 5 \cong \angle 3$
8. $m\angle 5 = m\angle 4$
9. $m\angle 5 + m\angle 6 = m\angle 3 + m\angle 4$
10. $m\angle APL + m\angle PAR = 180$
11. $m\angle 1 + m\angle 2 + m\angle 5 + m\angle 6 = 180$
12. Reword Theorem 3-8 as two statements, one describing existence and the other describing uniqueness.
13. Reword Theorem 3-9 as two statements, one describing existence and the other describing uniqueness.

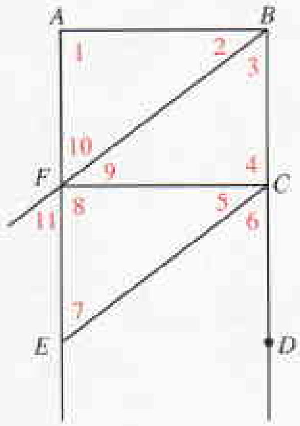


14. How many lines can be drawn through P parallel to $\overleftrightarrow{QR}$?
15. How many lines can be drawn through Q parallel to $\overleftrightarrow{PR}$?
16. How many lines can be drawn through P perpendicular to $\overleftrightarrow{QR}$?
17. In the plane of P, Q, and R, how many lines can be drawn through R perpendicular to $\overleftrightarrow{QR}$? What postulate or theorem justifies your answer?
18. In space, how many lines can be drawn through R perpendicular to $\overleftrightarrow{QR}$?
19. True or false?
    a. Two lines perpendicular to a third line must be parallel.
    b. In a plane two lines perpendicular to a third line must be parallel.
    c. In a plane two lines parallel to a third line must be parallel.
    d. Any two lines parallel to a third line must be parallel.
20. Use the diagram to explain why Theorem 3-10 is true for coplanar lines. That is, if $k \parallel l$ and $k \parallel n$, why does it follow that $l \parallel n$?



Written Exercises

In each exercise some information is given. Use this information to name the segments that must be parallel. If there are no such segments, write *none*.



A

1. $\angle 2 \cong \angle 9$

2. $\angle 6 \cong \angle 7$

3. $m\angle 1 = m\angle 8 = 90$

4. $\angle 5 \cong \angle 9$

5. $m\angle 2 = m\angle 5$

6. $\angle 3 \cong \angle 11$

7. $m\angle 1 = m\angle 4 = 90$

8. $m\angle 10 = m\angle 11$

9. $m\angle 8 + m\angle 5 + m\angle 6 = 180$

10. $\overline{FC} \perp \overline{AE}$ and $\overline{FC} \perp \overline{BD}$

11. $m\angle 5 + m\angle 6 = m\angle 9 + m\angle 10$

12. $\angle 7$ and $\angle EFB$ are supplementary.

13. $\angle 2$ and $\angle 3$ are complementary and $m\angle 1 = 90$.

14. $m\angle 2 + m\angle 3 = m\angle 4$

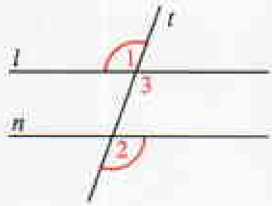
15. $m\angle 7 = m\angle 3 = m\angle 10$

16. $m\angle 4 = m\angle 8 = m\angle 1$

17. Write the reasons to complete the proof: If two lines are cut by a transversal and alternate exterior angles are congruent, then the lines are parallel.

Given: Transversal t cuts lines l and n;
$\angle 2 \cong \angle 1$

Prove: $l \parallel n$

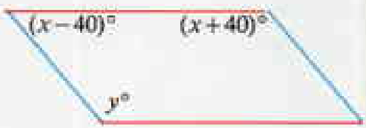


Proof:

Statements	Reasons
1. $\angle 2 \cong \angle 1$	1. ?
2. $\angle 1 \cong \angle 3$	2. ?
3. $\angle 2 \cong \angle 3$	3. ?
4. $l \parallel n$	4. ?

Find the values of x and y that make the red lines parallel *and* the blue lines parallel.

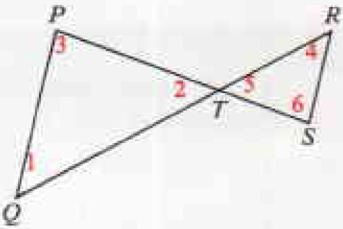
B **18.**



19.

20. Given: $\angle 1 \cong \angle 2$; $\angle 4 \cong \angle 5$
What can you prove about $\overline{PQ}$ and $\overline{RS}$? Be prepared to give your reasons in class, if asked.

21. Given: $\angle 3 \cong \angle 6$
What can you prove about other angles? Be prepared to give your reasons in class, if asked.



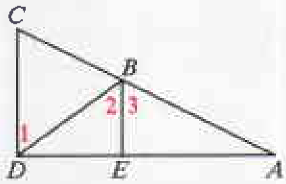
22. Copy what is shown for Theorem 3-6 on page 84. Then write a proof in two-column form.

23. Copy what is shown for Theorem 3-7 on page 84. Then write a proof in two-column form.

24. Given: $\overline{BE}$ bisects $\angle DBA$; $\angle 3 \cong \angle 1$
Prove: $\overline{CD} \parallel \overline{BE}$

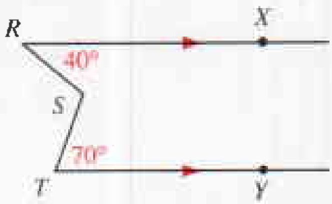
25. Given: $\overline{BE} \perp \overline{DA}$; $\overline{CD} \perp \overline{DA}$
Prove: $\angle 1 \cong \angle 2$

26. Given: $\angle C \cong \angle 3$; $\overline{BE} \perp \overline{DA}$
Prove: $\overline{CD} \perp \overline{DA}$

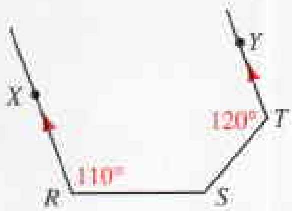


Find the measure of $\angle RST$. (*Hint*: Draw a line through S parallel to $\overline{RX}$ and $\overline{TY}$.)

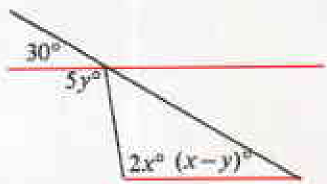
27.



28.



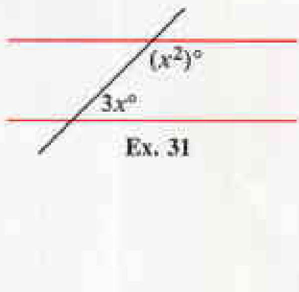
29. Find the values of x and y that make the lines shown in red parallel.



Ex. 29

C **30.** Draw two parallel lines cut by a transversal. Then draw the bisectors of two corresponding angles. What appears to be true about the bisectors? Prove that your conclusion is true.

31. Find the value of x that makes the lines shown in red parallel.

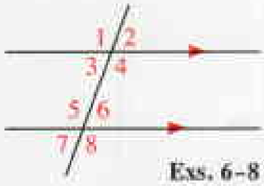


Ex. 31

Self-Test 1

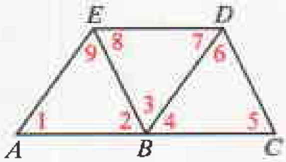
Complete each statement with the word *always*, *sometimes*, or *never*.

1. Two lines that do not intersect are __?__ parallel.
2. Two skew lines __?__ intersect.
3. Two lines parallel to a third line are __?__ parallel.
4. If a line is parallel to plane X and also to plane Y, then plane X and plane Y are __?__ parallel.
5. Plane X is parallel to plane Y. If plane Z intersects X in line l and Y in line n, then l is __?__ parallel to n.
6. Name two pairs of congruent alternate interior angles.
7. Name two pairs of congruent corresponding angles.
8. Name a pair of supplementary same-side interior angles.



Exs. 6–8

9. Complete: If $\overline{AE} \parallel \overline{BD}$, then $\angle 1 \cong$ __?__ and $\angle 9 \cong$ __?__.
10. If $\overline{ED} \parallel \overline{AC}$, name all pairs of angles that must be congruent.
11. If $\overline{ED} \parallel \overline{AC}$ and $\overline{EB} \parallel \overline{DC}$, name all angles that must be congruent to $\angle 5$.
12. Complete: If $\overline{ED} \parallel \overline{AC}$, $\overline{EB} \parallel \overline{DC}$, and $m\angle 2 = 65$, then $m\angle 8 =$ __?__ and $m\angle EDC =$ __?__.



Exs. 9–15

Use the given information to name the segments (if any) that must be parallel.

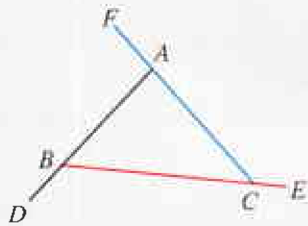
13. $\angle 3 \cong \angle 6$
14. $\angle 9 \cong \angle 6$
15. $m\angle 7 + m\angle AED = 180$

16. Complete: Through a point outside a line, __?__ line(s) can be drawn parallel to the given line, and __?__ line(s) can be drawn perpendicular to the given line.

Explorations

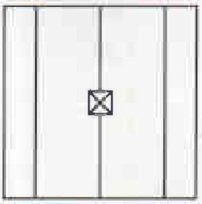
These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.

Draw a triangle ABC. At each vertex extend one side, as shown in the diagram. Measure all six angles formed. Repeat on several triangles. What do you notice?
What is the sum of the measures of the angles inside the triangle? of the angles outside the triangle?



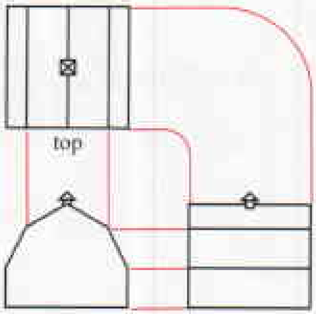
Application — *Technical Drawing*

Can the shape of a three-dimensional object be determined from a single two-dimensional image? For example, if you photographed the barn shown on page 75 from a point directly above, then your photograph might look something like the sketch shown at the right. You cannot tell from this one photograph how the roof slopes or anything about the sides of the barn. You would have a much better idea of the shape of the barn if you could also see it from the front and from one side.

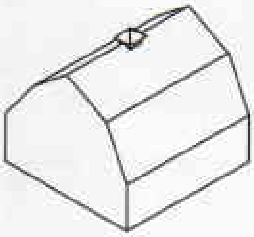


The three views of the barn are the parts of an *orthographic projection*, a set of projections of an object into three planes perpendicular to one another. They show the actual shape of the building much more clearly than any single picture.

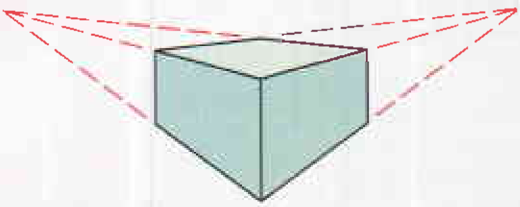
To make an orthographic projection of an object, draw a top view, a front view, and a side view. Arrange the three views in an "L" shaped pattern as illustrated in the figure on the left below. Some corresponding vertices have been connected with red lines.

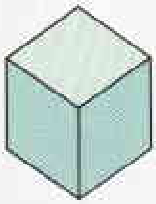


orthographic projection

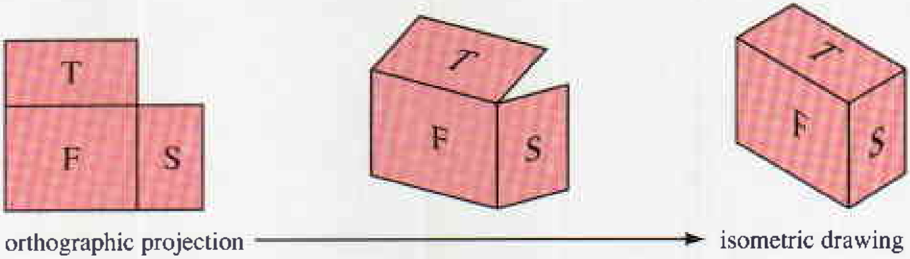

isometric drawing

A related method of representing a three-dimensional object by a two-dimensional image is an *isometric drawing*. In this type of representation the object is viewed at an angle that allows simultaneous vision of the top, front, and one side. Unlike most drawings, however, an isometric drawing does not show perspective. Rather, congruent sides are drawn congruent. Because we are accustomed to seeing objects in perspective, an isometric representation can appear distorted to us. The following figures illustrate the difference between a perspective drawing (left) and an isometric drawing (right).

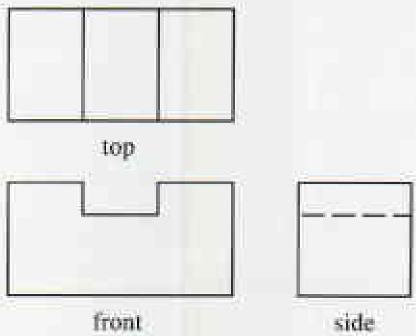




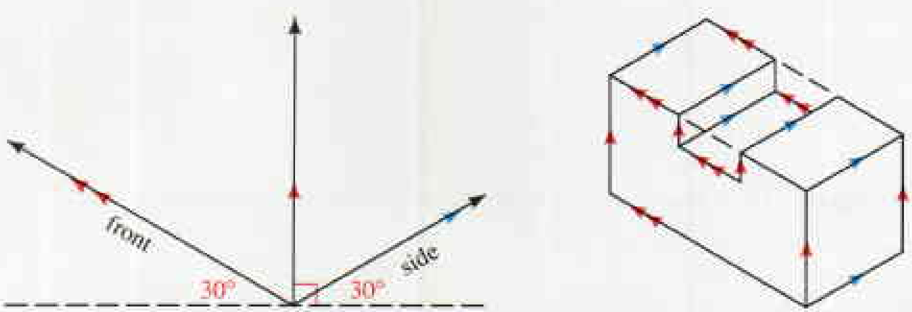
To make an orthographic projection into an isometric drawing, we fold the top, front, and side together. The base of the box becomes angled, but vertical lines remain vertical, parallel lines remain parallel, and congruence is preserved.



To illustrate, we shall make an isometric drawing of the solid whose orthographic projection is shown below. Visible edges and intersections are shown as solid lines. Hidden edges are shown as dashed lines.



Begin by drawing three rays with a common endpoint such that one ray is vertical and the other two rays are 30° off of horizontal. Mark off the lengths of the front, side, and height of the solid. By drawing congruent segments and by showing parallel edges as parallel in the figure, you can finish the isometric drawing. The figures that follow suggest the procedure.

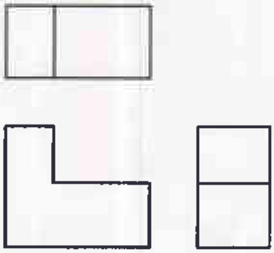


Industry requires millions of drawings similar to these each year. The drafters who make these drawings professionally combine the knowledge of parallel lines with the skills of using a compass, a protractor, and a ruler. More recently, drafters make drawings like these using a computer and a special printer.

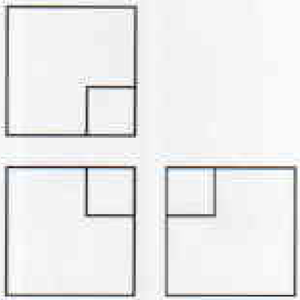
Exercises

Match the orthographic projections with their isometric drawings. If there is no isometric drawing, then make one.

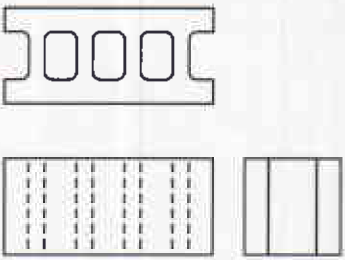
1.



2.



3.



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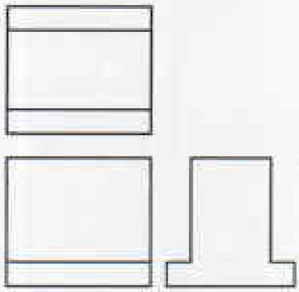

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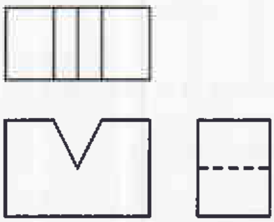

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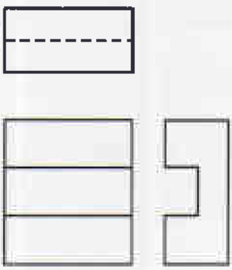
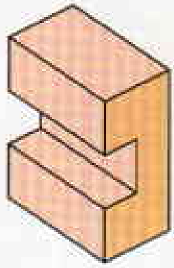
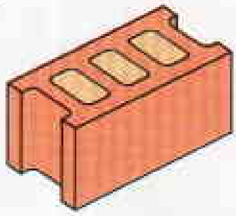


a.



b.



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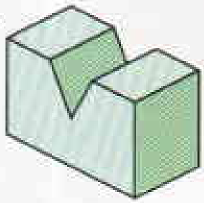

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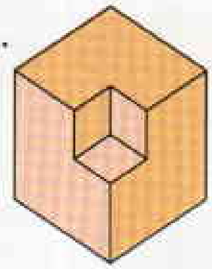
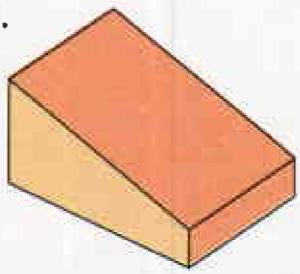


Trace each figure. Then make an orthographic projection of the figure.

7.



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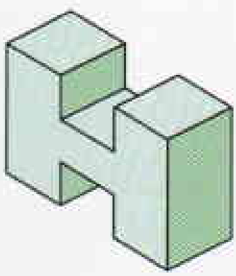


Applying Parallel Lines to Polygons

Objectives

1. Classify triangles according to sides and to angles.
2. State and apply the theorem and the corollaries about the sum of the measures of the angles of a triangle.
3. State and apply the theorem about the measure of an exterior angle of a triangle.
4. Recognize and name convex polygons and regular polygons.
5. Find the measures of interior angles and exterior angles of convex polygons.
6. Understand and use inductive reasoning.

3-4 *Angles of a Triangle*

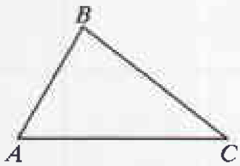
A **triangle** is the figure formed by three segments joining three noncollinear points. Each of the three points is a **vertex** of the triangle. (The plural of *vertex* is *vertices*.) The segments are the **sides** of the triangle.

Triangle ABC ($\triangle ABC$) is shown.

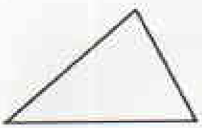
Vertices of $\triangle ABC$: points A, B, C

Sides of $\triangle ABC$: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

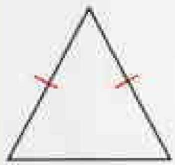
Angles of $\triangle ABC$: $\angle A$, $\angle B$, $\angle C$



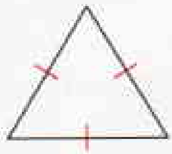
A triangle is sometimes classified by the number of congruent sides it has.



Scalene triangle
No sides congruent



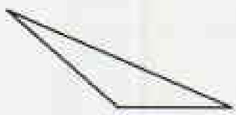
Isosceles triangle
At least two sides congruent



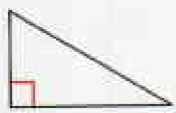
Equilateral triangle
All sides congruent

Triangles can also be classified by their angles.

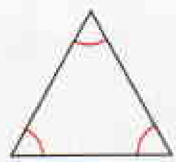
Acute △
Three acute ∠s



Obtuse △
One obtuse ∠



Right △
One right ∠



Equiangular △
All ∠s congruent

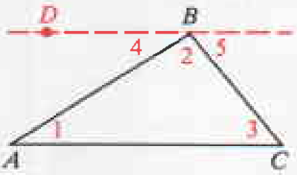
An **auxiliary line** is a line (or ray or segment) added to a diagram to help in a proof. An auxiliary line is used in the proof of the next theorem, one of the best-known theorems of geometry. The auxiliary line is shown as a dashed line in the diagram.

Theorem 3-11

The sum of the measures of the angles of a triangle is 180.

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$



Proof:

Statements	Reasons
1. Through B draw $\overleftrightarrow{BD}$ parallel to $\overleftrightarrow{AC}$.	1. Through a point outside a line, there is exactly one line $\parallel$ to the given line.
2. $m\angle DBC + m\angle 5 = 180$; $m\angle DBC = m\angle 4 + m\angle 2$	2. Angle Addition Postulate
3. $m\angle 4 + m\angle 2 + m\angle 5 = 180$	3. Substitution Property
4. $\angle 4 \cong \angle 1$, or $m\angle 4 = m\angle 1$; $\angle 5 \cong \angle 3$, or $m\angle 5 = m\angle 3$	4. If two parallel lines are cut by a transversal, then alt. int. $\measuredangle$s are $\cong$.
5. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	5. Substitution Property

A statement that can be proved easily by applying a theorem is often called a **corollary** of the theorem. Corollaries, like theorems, can be used as reasons in proofs. Each of the four statements that are shown below is a corollary of Theorem 3-11.

Corollary 1

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

Corollary 2

Each angle of an equiangular triangle has measure 60.

Corollary 3

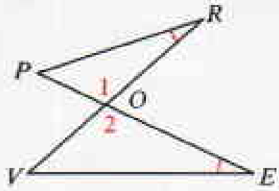
In a triangle, there can be at most one right angle or obtuse angle.

Corollary 4

The acute angles of a right triangle are complementary.

In the classroom exercises you will explain how these corollaries follow from Theorem 3-11.

Example 1 Is $\angle P \cong \angle V$?

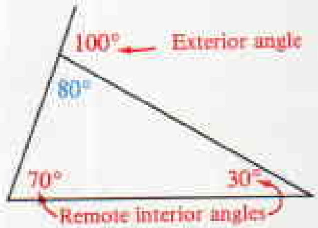


Solution $\angle R \cong \angle E$ (Given in diagram)

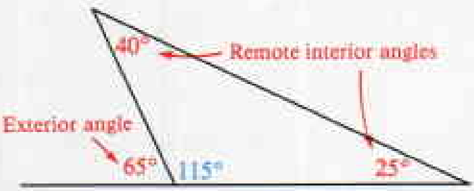
$\angle 1 \cong \angle 2$ (Vertical angles are congruent.)

Thus two angles of $\triangle PRO$ are congruent to two angles of $\triangle VEO$, and therefore $\angle P \cong \angle V$ by Corollary 1.

When one side of a triangle is extended, an *exterior angle* is formed as shown in the diagrams below. Because an exterior angle of a triangle is always a supplement of the adjacent interior angle of the triangle, its measure is related in a special way to the measure of the other two angles of the triangle, called the *remote interior angles.*



$100 = 70 + 30$



$65 = 40 + 25$

Theorem 3-12

The measure of an exterior angle of a triangle equals the sum of the measures of the two remote interior angles.

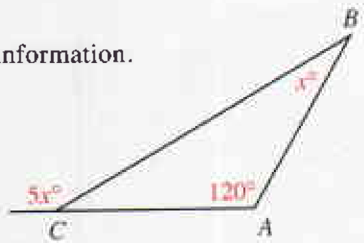
The proof of Theorem 3-12 is left as Classroom Exercise 15.

Example 2 In $\triangle ABC$, $m\angle A = 120$ and an exterior angle at C is five times as large as $\angle B$. Find $m\angle B$.

Solution Let $m\angle B = x$.

Draw a diagram that shows the given information.
Then apply Theorem 3-12.

$$\begin{aligned} 5x &= 120 + x \\ 4x &= 120 \\ x &= 30 \\ m\angle B &= 30 \end{aligned}$$



Classroom Exercises

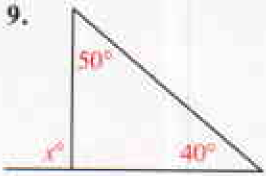
Complete each statement with the word *always, sometimes,* or *never.*

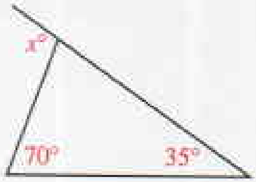
1. If a triangle is isosceles, then it is __?__ equilateral.
2. If a triangle is equilateral, then it is __?__ isosceles.
3. If a triangle is scalene, then it is __?__ isosceles.
4. If a triangle is obtuse, then it is __?__ isosceles.

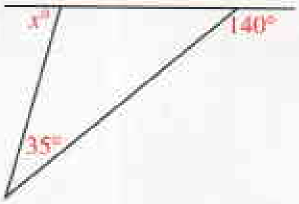
Explain how each corollary of Theorem 3-11 follows from the theorem.

5. Corollary 1
6. Corollary 2
7. Corollary 3
8. Corollary 4

Find the value of x.

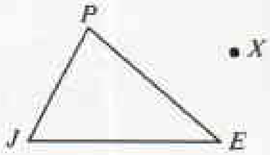
9. 

10. 

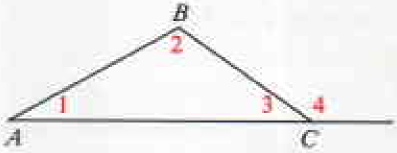
11. 

What is wrong with each of the following instructions?

12. Draw the bisector of $\angle J$ to the midpoint of $\overline{PE}$.
13. Draw the line from P perpendicular to $\overline{JE}$ at its midpoint.
14. Draw the line through P and X parallel to $\overleftrightarrow{JE}$.

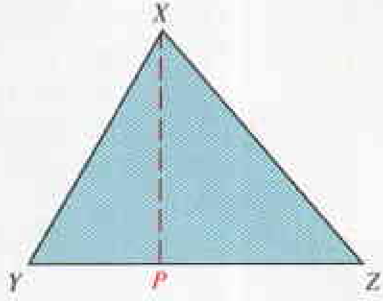


15. In the diagram you know that
(1) $m\angle 1 + m\angle 2 + m\angle 3 = 180$
(2) $m\angle 3 + m\angle 4 = 180$
Explain how these equations allow you to prove Theorem 3-12.



16. Fold a corner of a sheet of paper and then cut along the fold to get a right triangle. Let the right angle be $\angle C$. Fold each of the other two vertices so that they coincide with point C. What result of this section does this illustrate?

17. Cut out any large $\triangle XYZ$. (If the triangle has a longest side, let that side be $\overline{YZ}$.) Fold so that X lies on the fold line and Y falls on $\overrightarrow{YZ}$. Let P be the intersection of $\overline{YZ}$ and the fold line. Unfold. Now fold the paper so that Y coincides with P. Fold it twice more so that both X and Z coincide with P. What result of this section does this illustrate?



Written Exercises

Draw a triangle that satisfies the conditions stated. If no triangle can satisfy the conditions, write *not possible*.

A

1. **a.** An acute isosceles triangle
   b. A right isosceles triangle
   c. An obtuse isosceles triangle

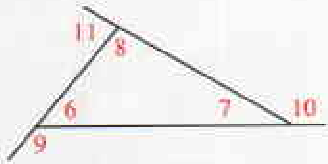
2. **a.** An acute scalene triangle
   b. A right scalene triangle
   c. An obtuse scalene triangle

3. A triangle with two acute exterior angles

4. A triangle with two obtuse exterior angles

Complete.

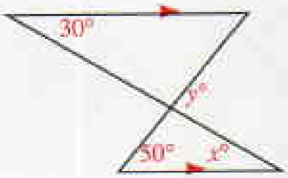
5. $m\angle 6 + m\angle 7 + m\angle 8 = \underline{\ ?\ }$.
6. If $m\angle 6 = 52$ and $m\angle 11 = 82$, then $m\angle 7 = \underline{\ ?\ }$.
7. If $m\angle 6 = 55$ and $m\angle 10 = 150$, then $m\angle 8 = \underline{\ ?\ }$.
8. If $m\angle 6 = x$, $m\angle 7 = x - 20$, and $m\angle 11 = 80$, then $x = \underline{\ ?\ }$.
9. If $m\angle 8 = 4x$, $m\angle 7 = 30$, and $m\angle 9 = 6x - 20$, then $x = \underline{\ ?\ }$.
10. $m\angle 9 + m\angle 10 + m\angle 11 = \underline{\ ?\ }$.



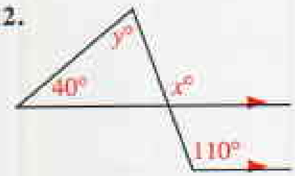
Exs. 5–10

Find the values of *x* and *y*.

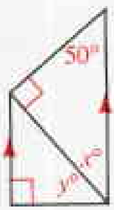
11.



12.

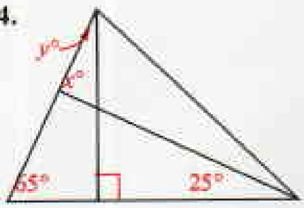


13.

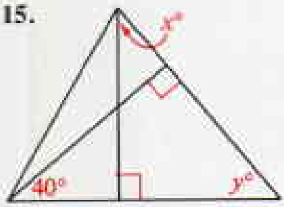


B

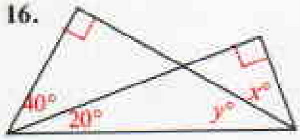
14.



15.



16.



17. The lengths of the sides of a triangle are $4n$, $2n + 10$, and $7n - 15$. Is there a value of n that makes the triangle equilateral? Explain.

18. The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$.
    a. Find the value(s) of t that make the triangle isosceles.
    b. Does any value of t make the triangle equilateral? Explain.

19. The largest two angles of a triangle are two and three times as large as the smallest angle. Find all three measures.

20. The measure of one angle of a triangle is 28 more than the measure of the smallest angle of the triangle. The measure of the third angle is twice the measure of the smallest angle. Find all three measures.

21. In $\triangle ABC$, $m\angle A = 60$ and $m\angle B < 60$. What can you say about $m\angle C$?

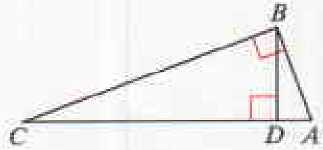
22. In $\triangle RST$, $m\angle R = 90$ and $m\angle S > 20$. What can you say about $m\angle T$?

23. Given: $\overline{AB} \perp \overline{BC}$; $\overline{BD} \perp \overline{AC}$

a. If $m\angle C = 22$, find $m\angle ABD$.

b. If $m\angle C = 23$, find $m\angle ABD$.

c. Explain why $m\angle ABD$ always equals $m\angle C$.

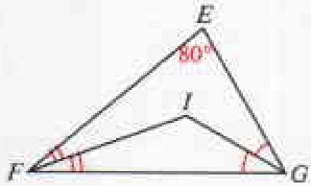


24. The bisectors of $\angle EFG$ and $\angle EGF$ meet at I.

a. If $m\angle EFG = 40$, find $m\angle FIG$.

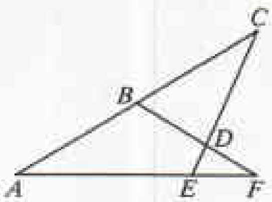
b. If $m\angle EFG = 50$, find $m\angle FIG$.

c. Generalize your results in (a) and (b).

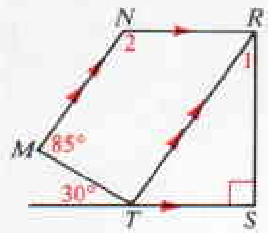


25. Given: $\angle ABD \cong \angle AED$

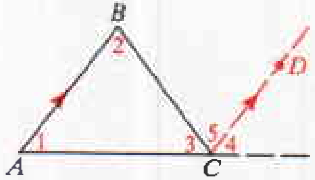
Prove: $\angle C \cong \angle F$



26. Find the measures of $\angle 1$ and $\angle 2$.

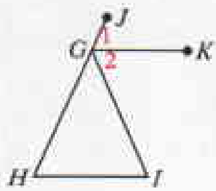


27. Prove Theorem 3-11 by using the diagram below. (Begin by stating what is given and what is to be proved. Draw the auxiliary ray shown.)



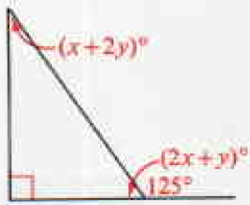
28. Given: $\overrightarrow{GK}$ bisects $\angle JGI$; $m\angle H = m\angle I$

Prove: $\overline{GK} \parallel \overline{HI}$

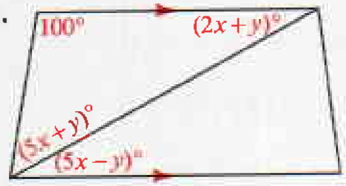


Find the values of x and y.

29.

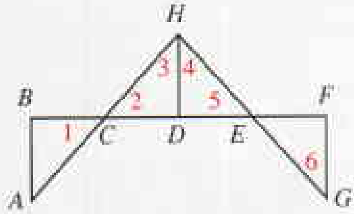


30.



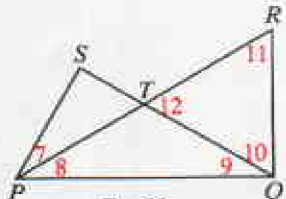
31. Given: $\overline{AB} \perp \overline{BF}$; $\overline{HD} \perp \overline{BF}$; $\overline{GF} \perp \overline{BF}$; $\angle A \cong \angle G$

Which numbered angles must be congruent?



C **32.** Given: $\overrightarrow{PR}$ bisects $\angle SPQ$; $\overline{PS} \perp \overline{SQ}$; $\overline{RQ} \perp \overline{PQ}$

Which numbered angles must be congruent?



Ex. 32

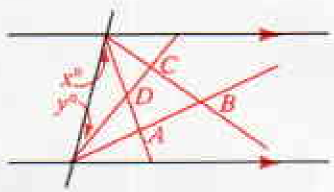
33. a. Draw two parallel lines and a transversal.

b. Use a protractor to draw bisectors of two same-side interior angles.

c. Measure the angles formed by the bisectors. What do you notice?

d. Prove your answer to part (c).

34. A pair of same-side interior angles are *trisected* (divided into three congruent angles) by the red lines in the diagram. Find out what you can about the angles of *ABCD*.



Explorations

These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.

Decide if the following statements are true or false. If you think the statement is true, give a convincing argument to support your belief. If you think the statement is false, make a sketch and give all the measurements of the triangle that you find as your counterexample. For each false statement, also discover if there are types of triangles for which the statement is true.

1. The measure of an exterior angle is greater than the measure of any interior angle of a triangle.
2. An exterior angle is always an obtuse angle.
3. An exterior angle and some interior angle are supplementary.
4. The sum of the measures of an exterior angle and the remote interior angles is 180.

Career

Carpenter

Carpenters work in all parts of the construction industry. A self-employed carpenter may work on relatively small-scale projects—for example, remodeling rooms or making other

alterations in existing houses or even building new single-family houses. As an employee of a large building contractor, a carpenter may be part of the work force building apartment or office complexes, stores, factories, and other major projects. Some carpenters are employed solely to provide maintenance to a large structure, where they do repairs and upkeep and make any alterations in the structure that are required.

Carpenters with adequate experience and expertise may become specialists in some skill of their own choice, for example, framing, interior finishing, or cabinet making. A carpenter who learns all aspects of the building industry thoroughly may decide to go into business as a general contractor, responsible for all work on an entire project.

Although some carpenters learn the trade through four-year apprenticeships, most learn on the job. These workers begin as laborers or as carpenters' helpers. While they work in these jobs they gradually acquire the skills necessary to become carpenters themselves. Carpenters must be able to measure accurately and to apply their knowledge of arithmetic, geometry, and informal algebra. They also benefit from being able to read and understand plans, blueprints, and charts.

3-5 *Angles of a Polygon*

The word **polygon** means "many angles." Look at the figures at the left below and note that each polygon is formed by coplanar segments (called *sides*) such that:

(1) Each segment intersects exactly two other segments, one at each endpoint.

(2) No two segments with a common endpoint are collinear.

Can you explain why each of the figures at the right above is *not* a polygon?

A **convex polygon** is a polygon such that no line containing a side of the polygon contains a point in the interior of the polygon. The outline of the state flag of Arizona, shown at the left below, is a convex polygon. At the right below is the state flag of Ohio, whose outline is a nonconvex polygon.

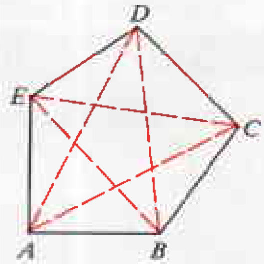
When we refer to a polygon in this book we will mean a convex polygon.

Polygons are classified according to the number of sides they have. Listed below are some of the special names for polygons you will see in this book.

Number of Sides	*Name*
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
8	octagon
10	decagon
n	n-gon

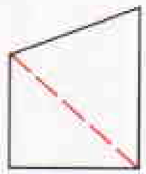
A triangle is the simplest polygon. The terms that we applied to triangles (such as *vertex* and *exterior angle*) also apply to other polygons.

When referring to a polygon, we list its consecutive vertices in order. Pentagon *ABCDE* and pentagon *BAEDC* are two of the many correct names for the polygon shown at the right.

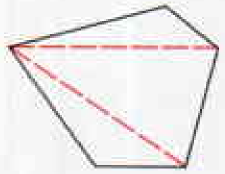


A segment joining two nonconsecutive vertices is a **diagonal** of the polygon. The diagonals of the pentagon at the right are indicated by dashes.

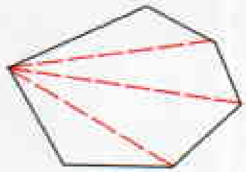
To find the sum of the measures of the angles of a polygon draw all the diagonals from just *one* vertex of the polygon to divide the polygon into triangles.



4 sides, 2 triangles
Angle sum = 2(180)



5 sides, 3 triangles
Angle sum = 3(180)



6 sides, 4 triangles
Angle sum = 4(180)

Note that the number of triangles formed in each polygon is two less than the number of sides. This result suggests the following theorem.

Theorem 3-13

The sum of the measures of the angles of a convex polygon with n sides is $(n - 2)180$.

Since the sum of the measures of the *interior* angles of a polygon depends on the number of sides, n, of the polygon, you would think that the same is true for the sum of the exterior angles. This is *not* true, as Theorem 3-14 reveals. The experiment suggested in Exercise 7 should help convince you of the truth of Theorem 3-14.

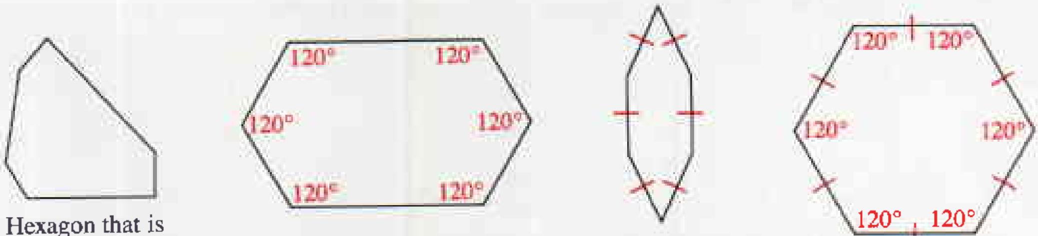
Theorem 3-14

The sum of the measures of the exterior angles of any convex polygon, one angle at each vertex, is 360.

Example 1 A polygon has 32 sides. Find (a) the sum of the measures of the interior angles and (b) the sum of the measures of the exterior angles, one angle at each vertex.

Solution (a) Interior angle sum = $(32 - 2)180 = 5400$ (Theorem 3-13)
(b) Exterior angle sum = 360 (Theorem 3-14)

Polygons can be equiangular or equilateral. If a polygon is both equiangular and equilateral, it is called a **regular polygon.**



Hexagon that is neither equiangular nor equilateral | Equiangular hexagon | Equilateral hexagon | Regular hexagon

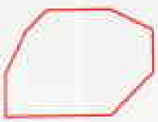
Example 2 A regular polygon has 12 sides. Find the measure of each interior angle.

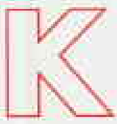
Solution 1 Interior angle sum = (12 − 2)180 = 1800
Each of the 12 congruent interior angles has measure 1800 ÷ 12, or 150.

Solution 2 Each exterior angle has measure 360 ÷ 12, or 30.
Each interior angle has measure 180 − 30, or 150.

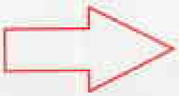
Classroom Exercises

Is the figure a convex polygon, a nonconvex polygon, or neither?

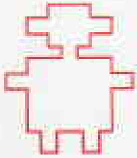
1. 

2. 

3. 

4. 

5. 

6. 

7. Imagine stretching a rubber band around each of the figures in Exercises 1–6. What is the relationship between the rubber band and the figure when the figure is a convex polygon?

8. A polygon has 102 sides. What is the interior angle sum? the exterior angle sum?

9. Complete the table for regular polygons.

Number of sides	6	10	20	?	?	?	?
Measure of each ext. ∠	?	?	?	10	20	?	?
Measure of each int. ∠	?	?	?	?	?	179	90

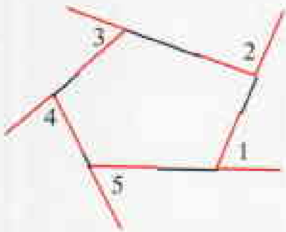
Written Exercises

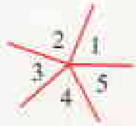
For each polygon, find (a) the interior angle sum and (b) the exterior angle sum.

A **1.** Quadrilateral **2.** Pentagon **3.** Hexagon

4. Octagon **5.** Decagon **6.** n-gon

7. Draw a pentagon with one exterior angle at each vertex. Cut out the exterior angles and arrange them so that they all have a common vertex, as shown at the far right. What is the sum of the measures of the exterior angles? Repeat the experiment with a hexagon. Do your results support Theorem 3-14?

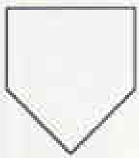




8. Complete the table for regular polygons.

Number of sides	9	15	30	?	?	?	?
Measure of each ext. ∠	?	?	?	6	8	?	?
Measure of each int. ∠	?	?	?	?	?	165	178

9. A baseball diamond's home plate has three right angles. The other two angles are congruent. Find their measure.



10. Four of the angles of a pentagon have measures 40, 80, 115, and 165. Find the measure of the fifth angle.

11. The face of a honeycomb consists of interlocking regular hexagons. What is the measure of each angle of these hexagons?

Sketch the polygon described. If no such polygon exists, write *not possible*.

12. A quadrilateral that is equiangular but not equilateral

13. A quadrilateral that is equilateral but not equiangular

14. A regular pentagon, one of whose angles has measure 120

15. A regular polygon, one of whose angles has measure 130

B **16.** The sum of the measures of the interior angles of a polygon is five times the sum of the measures of its exterior angles, one angle at each vertex. How many sides does the polygon have?

17. The measure of each interior angle of a regular polygon is eleven times that of an exterior angle. How many sides does the polygon have?

18. **a.** What is the measure of each interior angle of a regular pentagon?
b. Can you tile a floor with tiles shaped like regular pentagons? (Ignore the difficulty in tiling along the edges of the room.)

19. Make a sketch showing how to tile a floor using both squares and regular octagons.

20. The cover of a soccerball consists of interlocking regular pentagons and regular hexagons, as shown at the right. The second diagram shows that regular pentagons and hexagons cannot be interlocked in this pattern to tile a floor. Why not?


Possible


Impossible

21. In quadrilateral $ABCD$, $m\angle A = x$, $m\angle B = 2x$, $m\angle C = 3x$, and $m\angle D = 4x$. Find the value of x and then state which pair of sides of $ABCD$ must be parallel.

22. In pentagon $PQRST$, $m\angle P = 60$ and $m\angle Q = 130$. $\angle S$ and $\angle T$ are each three times as large as $\angle R$.
a. Find the measures of $\angle R$, $\angle S$, and $\angle T$.
b. Which pair of sides of $PQRST$ must be parallel?

23. $ABCDEFGHIJ$ is a regular decagon. If sides $\overline{AB}$ and $\overline{CD}$ are extended to meet at K, find the measure of $\angle K$.

24. $\overline{BC}$ is one side of a regular n-gon. The sides next to $\overline{BC}$ are extended to meet at W. Find the measure of $\angle W$ in terms of n.

25. The sum of the measures of the interior angles of a polygon is known to be between 2100 and 2200. How many sides does the polygon have?

C **26.** The sum of the measures of the interior angles of a polygon with n sides is S. Without using n in your answer, express in terms of S the sum of the measures of the angles of a polygon with:
a. $n + 1$ sides **b.** $2n$ sides

27. The formula $S = (n - 2)180$ can apply to nonconvex polygons if you allow the measure of an interior angle to be more than 180.
a. Illustrate this with a diagram that shows interior angles with measures greater than 180.
b. Does the reasoning leading up to Theorem 3-13 apply to your figure?

28. Given: The measure of each interior angle of a regular n-gon is x times that of an exterior angle.
a. Express x in terms of n.
b. For what values of n will x be an integer?

3-6 *Inductive Reasoning*

Throughout these first three chapters, we have been using deductive reasoning. Now we'll consider **inductive reasoning,** a kind of reasoning that is widely used in science and in everyday life.

Example 1 After picking marigolds for the first time, Connie began to sneeze. She also began sneezing the next four times she was near marigolds. Based on this past experience, Connie reasons inductively that she is allergic to marigolds.

Example 2 Every time Pitch has thrown a high curve ball to Slugger, Slugger has gotten a hit. Pitch concludes from this experience that it is not a good idea to pitch high curve balls to Slugger.

In coming to this conclusion, Pitch has used inductive reasoning. It may be that Slugger just happened to be lucky those times, but Pitch is too bright to feed another high curve to Slugger.

From these examples you can see how inductive reasoning differs from deductive reasoning.

Deductive Reasoning	Inductive Reasoning
Conclusion based on accepted statements (definitions, postulates, previous theorems, corollaries, and given information)	Conclusion based on several past observations
Conclusion *must* be true if hypotheses are true.	Conclusion is *probably* true, but not necessarily true.

Often in mathematics you can reason inductively by observing a pattern.

Example 3 Look for a pattern and predict the next number in each sequence.

a. 3, 6, 12, 24, __?__ **b.** 11, 15, 19, 23, __?__ **c.** 5, 6, 8, 11, 15, __?__

Solution **a.** Each number is twice the preceding number. The next number will be 2×24, or 48.

b. Each number is 4 more than the preceding number. The next number will be $23 + 4$, or 27.

c. Look at the differences between the numbers.

Numbers	5		6		8		11		15		?
Differences		1		2		3		4		?	

The next difference will be 5, and thus the next number will be $15 + 5$, or 20.

Classroom Exercises

Tell whether the reasoning process is deductive or inductive.

1. Ramon noticed that spaghetti had been on the school menu for the past five Wednesdays. Ramon decides that the school always serves spaghetti on Wednesday.
2. Ky did his assignment, adding the lengths of the sides of triangles to find the perimeters. Noticing the results for several equilateral triangles, he guesses that the perimeter of every equilateral triangle is three times the length of a side.
3. By using the definitions of equilateral triangle (a triangle with three congruent sides) and of perimeter (the sum of the lengths of the sides of a figure), Katie concludes that the perimeter of every equilateral triangle is three times the length of a side.
4. Linda observes that $(-1)^2 = +1$, $(-1)^4 = +1$, and $(-1)^6 = +1$. She concludes that every even power of (-1) is equal to $+1$.
5. John knows that multiplying a number by -1 merely changes the sign of the number. He reasons that multiplying a number by an even power of -1 will change the sign of the number an even number of times. He concludes that this is equivalent to multiplying a number by $+1$, so that every even power of -1 is equal to $+1$.
6. Look at the discussion leading up to the statement of Theorem 3-13 on page 102. Is the thinking inductive or deductive?

Written Exercises

Look for a pattern and predict the next two numbers in each sequence.

A

1. 1, 4, 16, 64, . . .
2. 18, 15, 12, 9, . . .
3. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$
4. 1, 4, 9, 16, . . .
5. 2, 3, 5, 8, 12, . . .
6. 10, 12, 16, 22, 30, . . .
7. 40, 39, 36, 31, 24, . . .
8. $8, -4, 2, -1, \frac{1}{2}, \ldots$
9. 2, 20, 10, 100, 50, . . .

Accept the two statements as given information. State a conclusion based on *deductive* reasoning. If no conclusion can be reached, write *none*.

10. Chan is older than Pedro.
    Pedro is older than Sarah.
11. Valerie is older than Greg.
    Dan is older than Greg.
12. Polygon G has more than 6 sides.
    Polygon G has fewer than 8 sides.
13. Polygon G has more than 6 sides.
    Polygon K has more than 6 sides.
14. There are three sisters. Two of them are athletes and two of them like tacos. Can you be sure that both of the athletes like tacos? Do you reason deductively or inductively to conclude the following? *At least one of the athletic sisters likes tacos.*

For each exercise, write the equation you think should come next. Check your prediction with a calculator.

15. $1 \times 9 + 2 = 11$
$12 \times 9 + 3 = 111$
$123 \times 9 + 4 = 1111$

16. $9 \times 9 + 7 = 88$
$98 \times 9 + 6 = 888$
$987 \times 9 + 5 = 8888$

17. $9^2 = 81$
$99^2 = 9801$
$999^2 = 998001$

Draw several diagrams to help you decide whether each statement is true or false. If it is false, show a counterexample. If it is true, draw and label a diagram you could use in a proof. List, in terms of the diagram, what is given and what is to be proved. Do *not* write a proof.

B **18.** If a triangle has two congruent sides, then the angles opposite those sides are congruent.

19. If a triangle has two congruent angles, then the sides opposite those angles are congruent.

20. If two triangles have equal perimeters, then they have congruent sides.

21. All diagonals of a regular pentagon are congruent.

22. If both pairs of opposite sides of a quadrilateral are parallel, then the diagonals bisect each other.

23. If the diagonals of a quadrilateral are congruent and also perpendicular, then the quadrilateral is a regular quadrilateral.

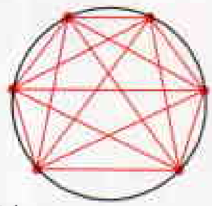
24. The diagonals of an equilateral quadrilateral are congruent.

25. The diagonals of an equilateral quadrilateral are perpendicular.

26. a. Study the diagrams below. Then guess the number of regions for the fourth diagram. Check your answer by counting.

2 points	3 points	4 points	5 points
2 regions	4 regions	8 regions	___?___ regions

b. Using 6 points on a circle as shown, guess the number of regions within the circle. Carefully check your answer by counting.



Important note: This exercise shows that a pattern predicted on the basis of a few cases may be incorrect. To be sure of a conclusion, use a deductive proof.

27. a. Draw several quadrilaterals whose opposite sides are parallel. With a protractor measure both pairs of opposite angles of each figure. On the basis of the diagrams and measurements, what do you guess is true for all such quadrilaterals? (*Note*: See Exercise 23, page 82.)

b. State and prove the converse of your conclusion about opposite angles in part (a).

c. Write a biconditional about pairs of opposite angles of a quadrilateral.

C 28. a. Substitute each of the integers from 1 to 9 for n in the expression $n^2 + n + 11$.

b. Using inductive reasoning, guess what kind of number you will get when you substitute any positive integer for n in the expression $n^2 + n + 11$.

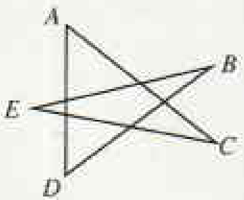
c. Test your guess by substituting 10 and 11 for n.

29. Complete the table for convex polygons.

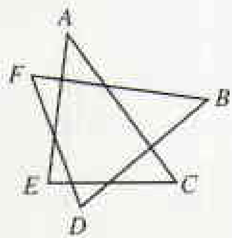
Number of sides	3	4	5	6	7	8	n
Number of diagonals	0	2	?	?	?	?	?

30. Find the sum of the measures of the angles formed at the tips of each star.

a. five-pointed star



b. six-pointed star



c. Using inductive reasoning, suggest a formula for the sum of the angle measures at the tips of an n-pointed star.

d. Using deductive reasoning, justify your formula.

♦ Calculator Key-In

Complete the right side of the first three equations in each exercise. Then use inductive reasoning to predict what the fourth equation would be if the pattern were continued. Check your prediction with your calculator.

1.
$1 \times 1 = \underline{?}$
$11 \times 11 = \underline{?}$
$111 \times 111 = \underline{?}$
$\underline{?} \times \underline{?} = \underline{?}$

2.
$6 \times 7 = \underline{?}$
$66 \times 67 = \underline{?}$
$666 \times 667 = \underline{?}$
$\underline{?} \times \underline{?} = \underline{?}$

3.
$8 \times 8 = \underline{?}$
$98 \times 98 = \underline{?}$
$998 \times 998 = \underline{?}$
$\underline{?} \times \underline{?} = \underline{?}$

4.
$7 \times 9 = \underline{?}$
$77 \times 99 = \underline{?}$
$777 \times 999 = \underline{?}$
$\underline{?} \times \underline{?} = \underline{?}$

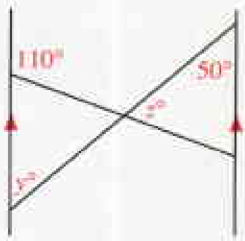
Self-Test 2

Complete.

1. If the measure of each angle of a triangle is less than 90, the triangle is called _?_.
2. If a triangle has no congruent sides, it is called _?_.
3. Each angle of an equiangular triangle has measure _?_.
4. In the diagram, m∠1 = _?_ and m∠2 = _?_.
5. If the measures of the acute angles of a right triangle are $2x + 4$ and $3x - 9$, then $x =$ _?_.

Ex. 4

6. Find the values of y and z.
7. The lengths of the sides of a triangle are $2x + 5$, $3x + 10$, and $x + 12$. Find all values of x that make the triangle isosceles.
8. An octagon has _?_ sides.
9. A regular polygon is both _?_ and _?_.
10. In a regular decagon, the sum of the measures of the exterior angles is _?_ and the measure of each interior angle is _?_.



Ex. 6

11. If the measure of each angle of a polygon is 174, then the measure of each exterior angle is _?_ and the polygon has _?_ sides.

Use inductive reasoning to predict the next number in each sequence.

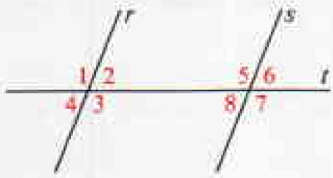
12. 2, −4, 8, −16, ...
13. 7, 12, 17, 22, 27, . . .
14. 1, 4, 9, 16, 25, . . .
15. 1, 4, 2, 8, 4, 16, 8, 32, . . .

Chapter Summary

1. Lines that do not intersect are either parallel or skew.
2. When two parallel lines are cut by a transversal:
   - **a.** corresponding angles are congruent;
   - **b.** alternate interior angles are congruent;
   - **c.** same-side interior angles are supplementary;
   - **d.** if the transversal is perpendicular to one of the two parallel lines, it is also perpendicular to the other one.
3. The chart on page 85 lists five ways to prove lines parallel.
4. Through a point outside a line, there is exactly one line parallel to, and exactly one line perpendicular to, the given line.
5. Two lines parallel to a third line are parallel to each other.

6. Triangles are classified (page 93) by the lengths of their sides and by the measures of their angles. In any $\triangle ABC$, $m\angle A + m\angle B + m\angle C = 180$.
7. The measure of an exterior angle of a triangle equals the sum of the measures of the two remote interior angles.
8. The sum of the measures of the angles of a convex polygon with n sides is $(n - 2)180$. The sum of the measures of the exterior angles, one angle at each vertex, is 360.
9. Polygons that are both equiangular and equilateral are regular polygons.
10. Inductive reasoning is the process of observing individual cases and then reaching a general conclusion suggested by them. The conclusion is probably, but not necessarily, true.

Chapter Review

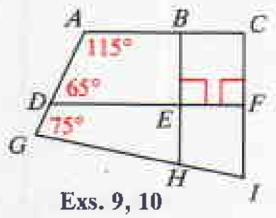


Exs. 1–7

1. $\angle 5$ and $\angle$ _?_ are same-side interior angles. 3–1
2. $\angle 5$ and $\angle 1$ are _?_ angles.
3. $\angle 5$ and $\angle 3$ are _?_ angles.
4. Line j, not shown, does not intersect line r. Must lines r and j be parallel?

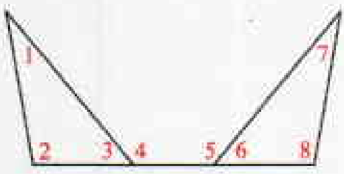
In the diagram above, $r \parallel s$.

5. If $m\angle 1 = 105$, then $m\angle 5 =$ _?_ and $m\angle 7 =$ _?_. 3–2
6. Solve for x: $m\angle 2 = 70$ and $m\angle 8 = 6x - 2$
7. Solve for y: $m\angle 3 = 8y - 40$ and $m\angle 8 = 2y + 20$
8. Lines a, b, and c are coplanar, $a \parallel b$, and $a \perp c$. What can you conclude? Explain.



Exs. 9, 10

9. Which line is parallel to $\overleftrightarrow{AB}$? Why? 3–3
10. Name a pair of parallel lines other than the pair in Exercise 9. Why must they be parallel?
11. Name five ways to prove two lines parallel.
12. If x and $2x - 15$ represent the measures of the acute angles of a right triangle, find the value of x. 3–4



Exs. 13–15

13. $m\angle 6 + m\angle 7 + m\angle 8 =$ _?_
14. If $m\angle 1 = 30$ and $m\angle 4 = 130$, then $m\angle 2 =$ _?_.
15. If $\angle 4 \cong \angle 5$ and $\angle 1 \cong \angle 7$, name two other pairs of congruent angles and give a reason for each answer.

16. a. Sketch a hexagon that is equiangular but not equilateral. 3–5
  b. What is its interior angle sum?
  c. What is its exterior angle sum?
17. A regular polygon has 18 sides. Find the measure of each interior angle.
18. A regular polygon has 24 sides. Find the measure of each exterior angle.
19. Each interior angle of a regular polygon has measure 150. How many sides does the polygon have?

Use inductive reasoning to predict the next two numbers in each sequence.

20. 15, 30, 45, 60, . . .

21. 100, -10, 1, $-\frac{1}{10}$, . . . 3–6

Chapter Test

Complete each statement with the word *always*, *sometimes*, or *never*.

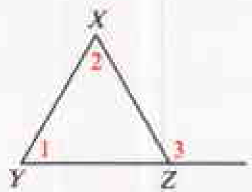
1. Two lines that have no points in common are __?__ parallel.
2. If a line is perpendicular to one of two parallel lines, then it is __?__ perpendicular to the other one.
3. If two lines are cut by a transversal and same-side interior angles are complementary, then the lines are __?__ parallel.
4. An obtuse triangle is __?__ a right triangle.
5. In $\triangle ABC$, if $\overline{AB} \perp \overline{BC}$, then $\overline{AC}$ is __?__ perpendicular to $\overline{BC}$.
6. As the number of sides of a regular polygon increases, the measure of each exterior angle __?__ decreases.

Find the value of *x*.

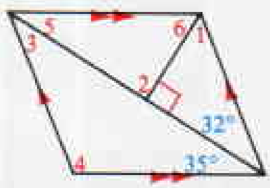
7. $m\angle 1 = 3x - 20$, $m\angle 2 = x$
8. $m\angle 2 = 2x + 12$, $m\angle 3 = 4(x - 7)$

Find the measures of the numbered angles.

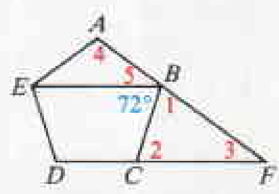
9. *XYZ* is regular.



10.

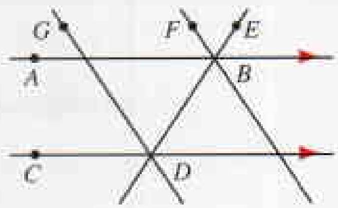


11. *ABCDE* is regular.



12. In the diagram for Exercise 11, explain why $\overline{EB}$ and $\overline{DF}$ must be parallel.

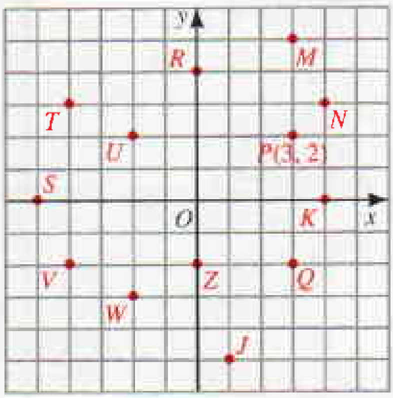
13. Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$; $\overrightarrow{BF}$ bisects $\angle ABE$;
$\overrightarrow{DG}$ bisects $\angle CDB$.
Prove: $\overleftrightarrow{BF} \parallel \overleftrightarrow{DG}$



14. Predict the next two numbers in the sequence 7, 9, 11, 13, ...

Algebra Review: *The Coordinate Plane*

1. What is the x-coordinate of point P?
2. What is the y-coordinate of point P?
3. What are the coordinates of the origin, point O?
4. Name the graph of the ordered pair $(0, -2)$.



Exs. 1–22

Name the coordinates of each point.

5. M **6.** N **7.** K **8.** R **9.** S
10. T **11.** U **12.** V **13.** W **14.** Q

15. Name all the points shown that lie on the x-axis.
16. Name all the points shown that lie on the y-axis.
17. What is the x-coordinate of every point that lies on a vertical line through P?
18. Which of the following points lie on a horizontal line through W?
a. $(-2, 1)$ **b.** $(2, 3)$ **c.** $(1, -3)$ **d.** $(-2, 0)$ **e.** $(0, -3)$ **f.** $(2, 0)$

Name all the points shown that lie in the quadrant indicated. (A point on an axis is not in any quadrant.)

19. Quadrant I **20.** Quadrant II **21.** Quadrant III **22.** Quadrant IV

Plot each point on graph paper.

23. O $(0, 0)$ **24.** A $(2, 1)$ **25.** B $(3, 4)$ **26.** C $(5, 0)$
27. D $(0, 3)$ **28.** E $(-3, 1)$ **29.** F $(-2, -1)$ **30.** G $(1, -2)$
31. H $(0, -4)$ **32.** I $(-4, 0)$ **33.** J $(4, -2)$ **34.** K $(-4, -3)$

Find the coordinates of the midpoint of $\overline{AB}$. (You may want to draw a diagram.)

35. A $(0, 1)$, B $(4, 1)$ **36.** A $(2, 0)$, B $(2, 10)$ **37.** A $(0, 1)$, B $(0, 5)$
38. A $(-3, 4)$, B $(-3, -4)$ **39.** A $(-5, -2)$, B $(-3, -2)$ **40.** A $(4, -1)$, B $(-2, -1)$

Cumulative Review: Chapters 1–3

Complete each statement with the word *always*, *sometimes*, or *never*.

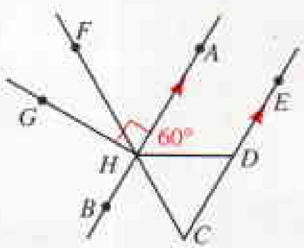
A

1. If $\overleftrightarrow{AB}$ intersects $\overline{CD}$, then $\overline{AB}$ _?_ intersects $\overline{CD}$.
2. If two planes intersect, their intersection is _?_ a line.
3. If $a \perp c$ and $b \perp c$, then a and b are _?_ parallel.
4. If two parallel planes are cut by a third plane, then the lines of intersection are _?_ coplanar.
5. A scalene triangle _?_ has an acute angle.

Draw a diagram that satisfies the conditions stated. If the conditions cannot be satisfied, write *not possible*.

6. $\overline{AB}$ and $\overline{XY}$ intersect and A is the midpoint of $\overline{XY}$.
7. A triangle is isosceles but not equilateral.
8. Three points all lie in both plane M and plane N.
9. Two lines intersect to form adjacent angles that are not supplementary.

10. Points A and B on a number line have coordinates -3.5 and 8.5. Find the coordinate of the midpoint of $\overline{AB}$.
11. $\overrightarrow{QX}$ bisects $\angle PQR$, $m\angle PQX = 5x + 13$, and $m\angle XQR = 9x - 39$. Find (a) the value of x and (b) $m\angle PQR$.
12. The measure of a supplement of an angle is 35 more than twice the complement of the angle. Find the measures of the angle, its supplement, and its complement.
13. The measures of two angles of a triangle are five and six times as large as the measure of the smallest angle. Find all three measures.



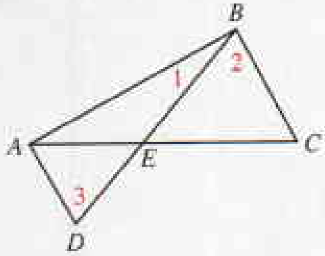
In the diagram $\overleftrightarrow{AB}$ bisects $\angle DHF$, $\overleftrightarrow{AB} \perp \overline{GH}$, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, and $m\angle AHD = 60$. Find the measure of each angle.

14. $\angle FHD$
15. $\angle AHG$
16. $\angle FHG$
17. $\angle GHB$
18. $\angle BHC$
19. $\angle DHC$
20. $\angle HDE$
21. $\angle HDC$
22. $\angle HCD$

Tell whether each statement is true or false. Then write the converse and tell whether it is true or false.

23. If two lines do not intersect, then they are parallel.
24. If two lines intersect to form right angles, then the lines are perpendicular.
25. An angle is acute only if it is not obtuse.
26. A triangle is isosceles if it is equilateral.

Name or state the postulate, definition, or theorem that justifies each statement about the diagram.



27. $\angle AED \cong \angle BEC$

28. $AE + EC = AC$

29. $m\angle 1 + m\angle 2 = m\angle ABC$

30. If $\angle 2 \cong \angle 3$, then $\overline{AD} \parallel \overline{BC}$.

31. $m\angle AEB = m\angle 2 + m\angle C$

32. If $\overline{DA} \perp \overline{AB}$, then $m\angle DAB = 90$.

33. $m\angle 1 + m\angle 3 + m\angle DAB = 180$

34. If $\angle ABC$ is a right angle, then $\overline{AB} \perp \overline{BC}$.

Complete.

35. The endpoint of $\overrightarrow{XY}$ is point __?__.

36. If the sum of the measures of two angles is 180, then the angles are __?__.

37. If the measure of each interior angle of a regular polygon is 108, then the polygon is a(n) __?__.

38. If M is the midpoint of $\overline{AB}$ and $AM = 12$, then $AB =$ __?__.

39. If two parallel lines are cut by a transversal, then alternate interior angles are __?__.

40. The process of forming a conclusion based on past observations or patterns is called __?__ reasoning.

41. When a statement and its converse are both true, they can be combined into one statement called a __?__.

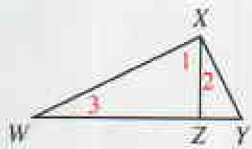
42. In a decagon the sum of the measures of the exterior angles is __?__.

43. In an octagon the sum of the measures of the interior angles is __?__.

44. Every triangle has at least two __?__ angles.

Write a two column proof.

B **45.** Given: $\overline{WX} \perp \overline{XY}$;
$\angle 1$ is comp. to $\angle 3$.
Prove: $\angle 2 \cong \angle 3$



46. Given: $\overline{RU} \parallel \overline{ST}$; $\angle R \cong \angle T$
Prove: $\overline{RS} \parallel \overline{UT}$