# 7 SIMILAR POLYGONS

**In mapping a landform like this river valley, cartographers must apply the geometric principles of similarity and draw accurately to scale.**

# Ratio, Proportion, and Similarity

## Objectives

1. Express a ratio in simplest form.
2. Solve for an unknown term in a given proportion.
3. Express a given proportion in an equivalent form.
4. State and apply the properties of similar polygons.

## 7-1 *Ratio and Proportion*

The **ratio** of one number to another is the quotient when the first number is divided by the second. This quotient is usually expressed in *simplest form*.

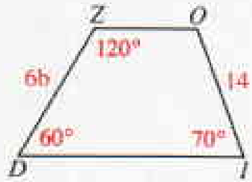
$$\text{The ratio of 8 to 12 is } \frac{8}{12}, \text{ or } \frac{2}{3}.$$

$$\text{If } y \neq 0, \text{ the ratio of } x \text{ to } y \text{ is } \frac{x}{y}.$$

Since we cannot divide by zero, a ratio $\frac{r}{s}$ is defined only if $s \neq 0$. When an expression such as $\frac{r}{s}$ appears in this book, you may assume that $s \neq 0$.

**Example 1** **a.** Find the ratio of $OI$ to $ZD$.

**b.** Find the ratio of the measure of the smallest angle of the trapezoid to that of the largest angle.




**Solution** **a.** $\frac{OI}{ZD} = \frac{14}{6b} = \frac{7}{3b}$

The ratio of $OI$ to $ZD$ is 7 to $3b$.

**b.** $\angle O$ has measure $180 - 70$, or 110. Thus $\angle D$ is the smallest angle and $\angle Z$ is the largest angle.

$$\frac{m\angle D}{m\angle Z} = \frac{60}{120} = \frac{1}{2}$$

The ratio of the measure of the smallest angle of the trapezoid to that of the largest angle is 1 to 2.

Ratios can be used to compare two numbers. To find the ratio of the lengths of two segments, the segments must be measured in terms of the same unit.

**Example 2** A poster is 1 m long and 52 cm wide. Find the ratio of the width to the length.

**Solution** *Method 1*

Use centimeters.
1 m = 100 cm
$$\frac{\text{width}}{\text{length}} = \frac{52}{100} = \frac{13}{25}$$

*Method 2*

Use meters.
52 cm = 0.52 m
$$\frac{\text{width}}{\text{length}} = \frac{0.52}{1} = \frac{52}{100} = \frac{13}{25}$$

Example 2 shows that the ratio of two quantities is not affected by the unit chosen.


Sometimes the ratio of $a$ to $b$ is written in the form $a:b$. This form can also be used to compare three or more numbers. The statement that three numbers are in the ratio $c:d:e$ (read "$c$ to $d$ to $e$") means:

(1) The ratio of the first two numbers is $c:d$.
(2) The ratio of the last two numbers is $d:e$.
(3) The ratio of the first and last numbers is $c:e$.

**Example 3** The measures of the three angles of a triangle are in the ratio 2:2:5. Find the measure of each angle.

**Solution** Let $2x$, $2x$, and $5x$ represent the measures.
$$2x + 2x + 5x = 180$$
$$9x = 180$$
$$x = 20$$
Then $2x = 40$ and $5x = 100$.

The measures of the angles are 40, 40, and 100.

A **proportion** is an equation stating that two ratios are equal. For example,

$$\frac{a}{b} = \frac{c}{d} \quad \text{and} \quad a:b = c:d$$

are equivalent forms of the same proportion. Either form can be read "$a$ is to $b$ as $c$ is to $d$." The number $a$ is called the first *term* of the proportion. The numbers $b$, $c$, and $d$ are the second, third, and fourth terms, respectively.

When three or more ratios are equal, you can write an *extended proportion:*

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

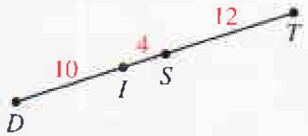
## Classroom Exercises

**Express the ratio in simplest form.**

1. $\frac{12}{20}$ 2. $\frac{3p}{5p}$ 3. $\frac{4n}{n^2}$ 4. $\frac{n^2}{4n}$

5. Is the ratio $a:b$ always, sometimes, or never equal to the ratio $b:a$? Explain.
6. An office copy machine can make a reduction to 90%, thus making the copy slightly smaller than the original. What is the ratio of the length of a line of text in the original to the length of a copy of that line?
7. Barbara is making oatmeal for breakfast. The instructions say to use 3 cups of water with 2 cups of oatmeal.
   a. What is the ratio of water to oatmeal?
   b. If Barbara uses 6 cups of water, how much oatmeal does she need?

**Express the ratio in simplest form.**




8. $DI:IS$ 9. $ST:DI$ 10. $IT:DT$
11. $DI:IT$ 12. $IT:DS$ 13. $IS:DI:IT$
14. What is the ratio of 750 mL to 1.5 L?
15. Can you find the ratio of 2 L to 4 km? Explain.
16. The ratio of the lengths of two segments is 4:3 when they are measured in centimeters. What is their ratio when they are measured in inches?
17. Three numbers aren't known, but the ratio of the numbers is 1:2:5. Is it possible that the numbers are 1, 2, and 5? 10, 20, and 50? 3, 6, and 20? $x$, $2x$, and $5x$?
18. What is the second term of the proportion $\frac{a}{b} = \frac{x}{y}$?

## Written Exercises

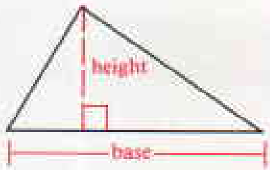
***ABCD* is a parallelogram. Find the value of each ratio.**


A
1. $AB:BC$ 2. $AB:CD$
3. $m\angle C:m\angle D$ 4. $m\angle B:m\angle C$
5. $AD$:perimeter of $ABCD$

**In Exercises 6–14, $x = 12$, $y = 10$, and $z = 24$. Write each ratio in simplest form.**

6. $x$ to $y$ 7. $z$ to $x$ 8. $x + y$ to $z$
9. $\frac{x}{x + z}$ 10. $\frac{x + y}{z + y}$ 11. $\frac{y + z}{x - y}$
12. $x:y:z$ 13. $z:x:y$ 14. $x:(x + y):(y + z)$

**Exercises 15–20 refer to a triangle. Express the ratio of the height to the base in simplest form.**




| | 15. | 16. | 17. | 18. | 19. | 20. |
|---|---|---|---|---|---|---|
| **height** | 5 km | 1 m | 0.6 km | 1 m | 8 cm | 40 mm |
| **base** | 45 km | 0.6 m | 0.8 km | 85 cm | 50 mm | 0.2 m |

**Write the algebraic ratio in simplest form.**

**21.** $\frac{3a}{4ab}$ **22.** $\frac{2cd}{5c^2}$ **23.** $\frac{3(x + 4)}{a(x + 4)}$

**In Exercises 24–29 find the measure of each angle.**

**B** **24.** The ratio of the measures of two complementary angles is 4:5.

**25.** The ratio of the measures of two supplementary angles is 11:4.

**26.** The measures of the angles of a triangle are in the ratio 3:4:5.

**27.** The measures of the acute angles of a right triangle are in the ratio 5:7.

**28.** The measures of the angles of an isosceles triangle are in the ratio 3:3:2.

**29.** The measures of the angles of a hexagon are in the ratio 4:5:5:8:9:9.

**30.** The perimeter of a triangle is 132 cm and the lengths of its sides are in the ratio 8:11:14. Find the length of each side.

**31.** The measures of the consecutive angles of a quadrilateral are in the ratio 5:7:11:13. Find the measure of each angle, draw a quadrilateral that satisfies the requirements, and explain why two sides must be parallel.

**32.** What is the ratio of the measure of an interior angle to the measure of an exterior angle in a regular hexagon? A regular decagon? A regular $n$-gon?

**33.** A team's best hitter has a lifetime batting average of .320. He has been at bat 325 times.

**a.** How many hits has he made?

**b.** The same player goes into a slump and doesn't get any hits at all in his next ten times at bat. What is his current batting average to the nearest thousandth?

**C** **34.** A basketball player has made 24 points out of 30 free throws. She hopes to make all her next free throws until her free-throw percentage is 85 or better. How many consecutive free throws will she have to make?

**35.** Points $B$ and $C$ lie on $\overline{AD}$. Find $AC$ if

$\frac{AB}{BD} = \frac{3}{4}$, $\frac{AC}{CD} = \frac{5}{6}$, and $BD = 66$.

**36.** Find the ratio of $x$ to $y$: $\frac{4}{y} + \frac{3}{x} = 44$

$$\frac{12}{y} - \frac{2}{x} = 44$$

# 7-2 Properties of Proportions

The first and last terms of a proportion are called the *extremes*. The middle terms are the *means*. In the proportions below, the extremes are shown in red. The means are shown in black.

$$a:b = c:d \qquad 6:9 = 2:3 \qquad \frac{6}{9} = \frac{2}{3}$$

Notice that $6 \cdot 3 = 9 \cdot 2$. This illustrates a property of all proportions, called the *means-extremes* property of proportions:

The product of the extremes equals the product of the means.

$\frac{a}{b} = \frac{c}{d}$ is *equivalent* to $ad = bc$.

The two equations are equivalent because we can change either of them into the other by multiplying (or dividing) each side by $bd$. Try this yourself.

It is often necessary to replace one proportion by an equivalent proportion. When you do so in a proof, you can use the reason "A property of proportions." The following properties will be justified in the exercises.

## Properties of Proportions

1. $\frac{a}{b} = \frac{c}{d}$ is equivalent to:

   a. $ad = bc$ b. $\frac{a}{c} = \frac{b}{d}$ c. $\frac{b}{a} = \frac{d}{c}$ d. $\frac{a+b}{b} = \frac{c+d}{d}$

2. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$, then $\frac{a+c+e+\cdots}{b+d+f+\cdots} = \frac{a}{b} = \cdots$.

**Example** Use the proportion $\frac{x}{y} = \frac{5}{2}$ to complete each statement.

a. $5y = \underline{\ ?\ }$ b. $\frac{x+y}{y} = \frac{?}{?}$

c. $\frac{2}{5} = \frac{?}{?}$ d. $\frac{x}{5} = \frac{?}{?}$

**Solution** a. $5y = 2x$ b. $\frac{x+y}{y} = \frac{7}{2}$

c. $\frac{2}{5} = \frac{y}{x}$ d. $\frac{x}{5} = \frac{y}{2}$

## Classroom Exercises

1. If $\frac{e}{f} = \frac{g}{h}$, which equation is correct?

   a. $ef = gh$ b. $eh = fg$ c. $eg = fh$

2. Which proportions are equivalent to $\frac{x}{12} = \frac{3}{4}$?

   a. $\frac{x}{3} = \frac{12}{4}$ b. $\frac{x}{4} = \frac{12}{3}$ c. $\frac{12}{x} = \frac{4}{3}$ d. $\frac{x + 12}{12} = \frac{7}{4}$

**Complete the statement.**

3. If $\frac{a}{b} = \frac{2}{3}$, then $3a = \underline{\ ?\ }$.

4. If $\frac{c}{d} = \frac{4}{7}$, then $\frac{d}{c} = \frac{?}{?}$.

5. If $\frac{e}{f} = \frac{5}{9}$, then $\frac{e}{5} = \frac{?}{?}$.

6. If $\frac{g}{h} = \frac{j}{8}$, then $\frac{j}{g} = \frac{?}{?}$.

7. If $\frac{k}{m} = \frac{2}{3}$, then $\frac{k + m}{m} = \frac{?}{?}$.

8. If $\frac{n}{p} = \frac{q}{r} = \frac{7}{9}$, then $\frac{n + q + 7}{p + r + 9} = \frac{?}{?}$.

9. a. Apply the means-extremes property of proportions to the proportion $\frac{e}{f} = \frac{g}{5}$ and you get $5e = \underline{\ ?\ }$.

   b. Apply the property to the proportion $\frac{5}{f} = \frac{g}{e}$ and you get $\underline{\ ?\ } = \underline{\ ?\ }$.

   c. Are the proportions $\frac{e}{f} = \frac{g}{5}$ and $\frac{5}{f} = \frac{g}{e}$ equivalent? Why?

10. Explain an easy way to show that the proportions $\frac{x}{7} = \frac{2}{3}$ and $\frac{x}{2} = \frac{3}{7}$ are not equivalent.

11. Apply the means-extremes property to $\frac{x}{10} = \frac{4}{5}$ and you get $5x = \underline{\ ?\ }$ and $x = \underline{\ ?\ }$.

12. If $\frac{4}{y} = \frac{7}{9}$, then $\underline{\ ?\ } = \underline{\ ?\ }$ and $y = \underline{\ ?\ }$.

**What can you conclude from the given information?**

13. $\frac{b}{a} = \frac{t}{x}$ and $\frac{a}{b} = \frac{x}{p}$

14. $\frac{2}{5} = \frac{y}{k}$ and $\frac{2}{z} = \frac{5}{k}$

15. Apply the means-extremes property to $\frac{a}{b} = \frac{c}{d}$ and also to $\frac{a}{c} = \frac{b}{d}$. (Note that you have justified Property 1(b) on page 245 by showing that each proportion is equivalent to the same equation.)

16. Explain why $\frac{a}{b} = \frac{c}{d}$ and $\frac{b}{a} = \frac{d}{c}$ are equivalent. (This justifies Property 1(c) on page 245.)

## Written Exercises

**Complete each statement.**

**A**

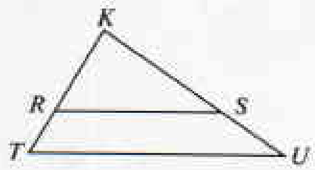
1. If $\frac{x}{3} = \frac{2}{5}$, then $5x = \underline{\ ?\ }$.
2. If $\frac{4}{x} = \frac{2}{7}$, then $2x = \underline{\ ?\ }$.
3. If $a:3 = 7:4$, then $4a = \underline{\ ?\ }$.
4. If $4:t = 8:9$, then $8t = \underline{\ ?\ }$.
5. If $\frac{a}{4} = \frac{b}{7}$, then $\frac{a}{b} = \frac{?}{?}$.
6. If $\frac{x}{y} = \frac{3}{8}$, then $\frac{y}{x} = \frac{?}{?}$.
7. If $\frac{x}{2} = \frac{y}{3}$, then $\frac{x+2}{2} = \underline{\ ?\ }$.
8. If $\frac{a}{b} = \frac{5-x}{x}$, then $\frac{a+b}{b} = \underline{\ ?\ }$.

**Find the value of x.**

9. $\frac{x}{4} = \frac{3}{5}$
10. $\frac{4}{x} = \frac{2}{5}$
11. $\frac{2}{5} = \frac{3x}{7}$
12. $\frac{8}{x} = \frac{2}{5}$
13. $\frac{x+5}{4} = \frac{1}{2}$
14. $\frac{x+3}{2} = \frac{4}{3}$
15. $\frac{x+2}{x+3} = \frac{4}{5}$
16. $\frac{2x+1}{4x-1} = \frac{2}{3}$
17. $\frac{x+3}{2} = \frac{2x-1}{3}$
18. $\frac{x+4}{x-4} = \frac{6}{5}$
19. $\frac{7}{6x-4} = \frac{9}{4x+6}$
20. $\frac{3x+5}{3} = \frac{18x+5}{7}$

**For the figure shown, it is given that $\frac{KR}{RT} = \frac{KS}{SU}$. Copy and complete the table.**

| | KR | RT | KT | KS | SU | KU |
|---|---|---|---|---|---|---|
| 21. | 12 | 9 | ? | 16 | ? | ? |
| 22. | 8 | ? | 10 | 12 | ? | ? |
| 23. | 16 | ? | ? | ? | 10 | 30 |
| 24. | ? | 2 | ? | 9 | ? | 12 |
| **B** 25. | ? | ? | 12 | 10 | 5 | ? |
| 26. | 12 | 4 | ? | ? | ? | 20 |
| 27. | ? | 9 | 36 | ? | ? | 48 |
| 28. | ? | ? | 30 | 28 | ? | 42 |




(*Hint for Ex. 25*: Let $KR = x$, then $RT = 12 - x$.)

29. Show that the proportions $\frac{a+b}{b} = \frac{c+d}{d}$ and $\frac{a}{b} = \frac{c}{d}$ are equivalent. (Note that this exercise justifies property 1(d) on page 245.)
30. Given the proportions $\frac{x+y}{y} = \frac{r}{s}$ and $\frac{x-y}{x+y} = \frac{s}{y}$, what can you conclude?

**Show that the given proportions are equivalent.**

31. $\frac{a - b}{a + b} = \frac{c - d}{c + d}$ and $\frac{a}{b} = \frac{c}{d}$

32. $\frac{a + c}{b + d} = \frac{a - c}{b - d}$ and $\frac{a}{b} = \frac{c}{d}$

**Find the value of $x$.**

33. $\frac{x}{x + 5} = \frac{x - 4}{x}$

34. $\frac{x - 2}{x} = \frac{x}{x + 3}$

35. $\frac{x + 1}{x - 2} = \frac{x + 5}{x - 6}$

C 36. $\frac{x - 1}{x - 2} = \frac{x + 4}{x + 2}$

37. $\frac{x(x + 5)}{4x + 4} = \frac{9}{5}$

38. $\frac{x - 1}{x + 2} = \frac{10}{3x - 2}$

**Find the values of $x$ and $y$.**

39. $\frac{y}{x - 9} = \frac{4}{7}$
$\frac{x + y}{x - y} = \frac{5}{3}$

40. $\frac{x - 3}{4} = \frac{y + 2}{2}$
$\frac{x + y - 1}{6} = \frac{x - y + 1}{5}$

41. Prove: If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a + c + e}{b + d + f} = \frac{a}{b}$. (*Hint*: Let $\frac{a}{b} = r$. Then $a = br$, $c = dr$, and $e =$ __?__.)

42. Explain how to extend the proof of Exercise 41 to justify Property 2 on page 245.

43. If $\frac{4a - 9b}{4a} = \frac{a - 2b}{b}$, find the numerical value of the ratio $a:b$.

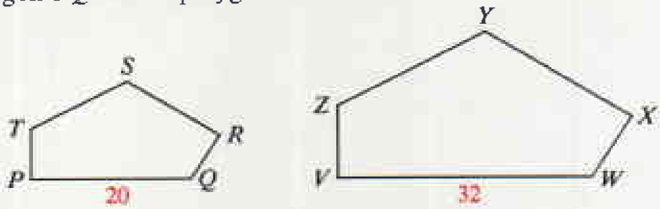
# 7-3 *Similar Polygons*

When you draw a diagram of a soccer field, you don't need an enormous piece of paper. You use a convenient sheet and draw *to scale*. That is, you show the right shape, but in a convenient size. Two figures, such as those below, that have the same shape are called *similar*.


Two polygons are **similar** if their vertices can be paired so that:

(1) Corresponding angles are congruent.
(2) Corresponding sides are in proportion. (Their lengths have the same ratio.)

When you refer to similar polygons, their corresponding vertices must be listed in the same order. If polygon *PQRST* is similar to polygon *VWXYZ*, you write polygon *PQRST* ~ polygon *VWXYZ*.




From the definition of similar polygons, we have:

(1) $\angle P \cong \angle V \quad \angle Q \cong \angle W \quad \angle R \cong \angle X \quad \angle S \cong \angle Y \quad \angle T \cong \angle Z$

(2) $\frac{PQ}{VW} = \frac{QR}{WX} = \frac{RS}{XY} = \frac{ST}{YZ} = \frac{TP}{ZV}$

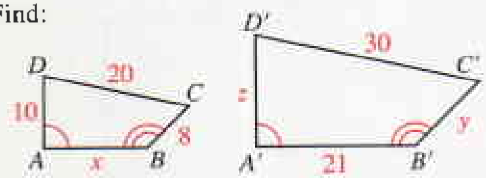
Similarity has some of the same properties as equality and congruence (page 37). Similarity is reflexive, symmetric, and transitive.

If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor** of the similarity. The scale factor of pentagon *PQRST* to pentagon *VWXYZ* is $\frac{PQ}{VW} = \frac{20}{32} = \frac{5}{8}$.

The example that follows shows one convenient way to label corresponding vertices: *A* and *A'* (read *A* prime), *B* and *B'*, and so on.

**Example** Quad. *ABCD* ~ quad. *A'B'C'D'*. Find:

**a.** their scale factor
**b.** the values of *x*, *y*, and *z*
**c.** the perimeters of the two quadrilaterals
**d.** the ratio of the perimeters




**Solution** **a.** The scale factor is $\frac{DC}{D'C'} = \frac{20}{30} = \frac{2}{3}$.

**b.** $\frac{DC}{D'C'} = \frac{AB}{A'B'}$ $\qquad$ $\frac{DC}{D'C'} = \frac{BC}{B'C'}$ $\qquad$ $\frac{DC}{D'C'} = \frac{AD}{A'D'}$

$\frac{2}{3} = \frac{x}{21}$ $\qquad$ $\frac{2}{3} = \frac{8}{y}$ $\qquad$ $\frac{2}{3} = \frac{10}{z}$

$x = 14$ $\qquad$ $y = 12$ $\qquad$ $z = 15$

**c.** The perimeter of quad. *ABCD* is 10 + 20 + 8 + 14 = 52.
The perimeter of quad. *A'B'C'D'* is 15 + 30 + 12 + 21 = 78.

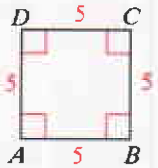
**d.** The ratio of the perimeters is $\frac{52}{78}$, or $\frac{2}{3}$.

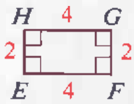
If you compare the ratio of the perimeters with the scale factor of the similarity, you discover they are the same. This property will be discussed further in Exercise 23 on page 251 and in Theorem 11-7.

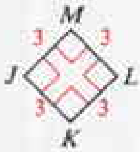
## Classroom Exercises

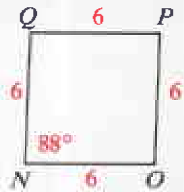
**Are the quadrilaterals similar? If they aren't, tell why not.**

1. *ABCD* and *EFGH*
2. *ABCD* and *JKLM*
3. *ABCD* and *NOPQ*
4. *JKLM* and *NOPQ*

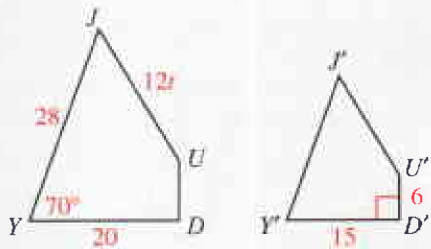



5. If the corresponding angles of two polygons are congruent, must the polygons be similar?
6. If the corresponding sides of two polygons are in proportion, must the polygons be similar?
7. Two polygons are similar. Do they have to be congruent?
8. Two polygons are congruent. Do they have to be similar?
9. Are all regular pentagons similar?
10. Quad. *JUDY* ~ quad. *J'U'D'Y'*. Complete.
    a. $m\angle Y'$ = __?__ and $m\angle D$ = __?__.
    b. The scale factor of quad. *JUDY* to quad. *J'U'D'Y'* is __?__.
    c. Find *DU*, *Y'J'*, and *J'U'*.
    d. The ratio of the perimeters is __?__.
    e. Explain why it is not true that quad. *DUJY* ~ quad. *Y'J'U'D'*.



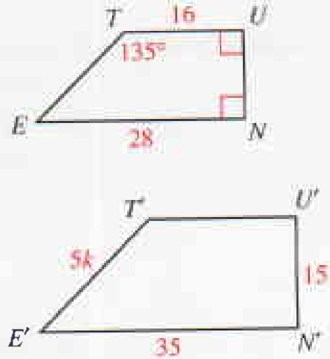

## Written Exercises

**Tell whether the two polygons are *always*, *sometimes*, or *never* similar.**

**A**

1. Two equilateral triangles
2. Two right triangles
3. Two isosceles triangles
4. Two scalene triangles
5. Two squares
6. Two rectangles
7. Two rhombuses
8. Two isosceles trapezoids
9. Two regular hexagons
10. Two regular polygons
11. A right triangle and an acute triangle
12. An isosceles triangle and a scalene triangle
13. A right triangle and a scalene triangle
14. An equilateral triangle and an equiangular triangle

**In Exercises 15-23 quad. $TUNE \sim$ quad. $T'U'N'E'$.**




**15.** What is the scale factor of quad. $TUNE$ to quad. $T'U'N'E'$?

**16.** What special kind of quadrilateral must quad. $T'U'N'E'$ be? Explain.

**17.** Find $m\angle T'$.

**18.** Find $m\angle E'$.

**19.** Find $UN$.

**20.** Find $T'U'$.

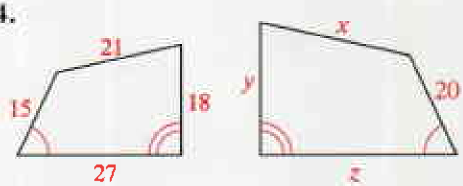
**21.** Find $TE$.

**22.** Find the ratio of the perimeters.

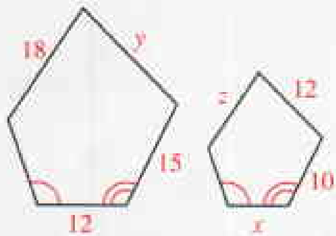
**B** **23.** What property of proportions on page 245 would you use to show that the ratio of the perimeters is equal to the ratio of the lengths of any two corresponding sides?

**Two similar polygons are shown. Find the values of $x$, $y$, and $z$.**

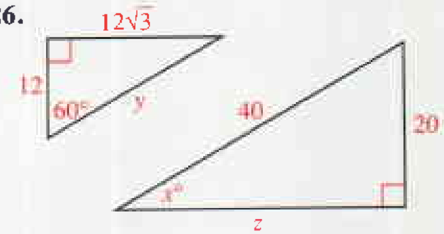
**24.**



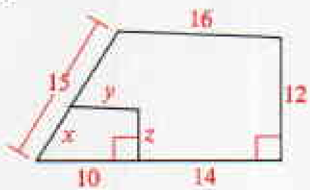

**25.**




**26.**




**27.**




**28.** Draw two equilateral hexagons that are clearly not similar.

**29.** Draw two equiangular hexagons that are clearly not similar.

**30.** If $\triangle ABC \sim \triangle DEF$, express $AB$ in terms of other lengths. (There are two possible answers.)

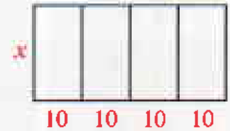
**31.** Explain how you can tell at once that quadrilateral $RSWX$ is not similar to quadrilateral $RSYZ$.


**Plot the given points on graph paper. Draw quadrilateral $ABCD$ and $\overline{A'B'}$. Locate points $C'$ and $D'$ so that $A'B'C'D'$ is similar to $ABCD$.**

**32.** $A(0, 0)$, $B(4, 0)$, $C(2, 4)$, $D(0, 2)$, $A'(-10, -2)$, $B'(-2, -2)$

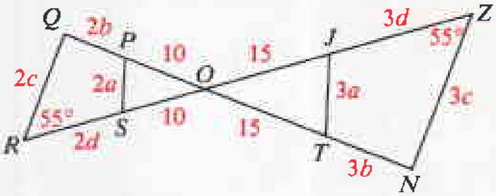
**33.** $A(0, 0)$, $B(4, 0)$, $C(2, 4)$, $D(0, 2)$, $A'(7, 2)$, $B'(7, 0)$

**34.** The card shown was cut into four congruent pieces with each piece similar to the original. Find the value of $x$.




**35.** Quad. $WHAT$ is a figure such that $WHAT \sim HATW$. Find the measure of each angle. What special kind of figure must the quadrilateral be?

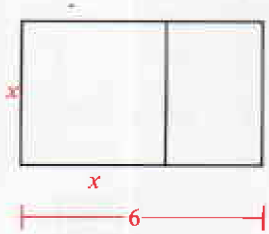
**C** **36.** What can you deduce from the diagram shown at the right? Explain.




**37.** The large rectangle shown is a *golden rectangle*. This means that when a square is cut off, the rectangle that remains is similar to the original rectangle.

**a.** How wide is the original rectangle?

**b.** The ratio of length to width in a golden rectangle is called the *golden ratio*. Write the golden ratio in simplified radical form. Then use a calculator to find an approximation to the nearest hundredth.




# Self-Test 1

**Express the ratio in simplest form.**

**1.** 9:15 **2.** 60 cm to 2 m **3.** $\frac{4ab}{6b^2}$

**Solve for $x$.**

**4.** $\frac{x}{8} = \frac{9}{12}$ **5.** $\frac{x-2}{2} = \frac{x+6}{4}$ **6.** $\frac{x}{5-x} = \frac{12}{8}$

**Tell whether the equation is equivalent to the proportion $\frac{a}{b} = \frac{5}{7}$.**

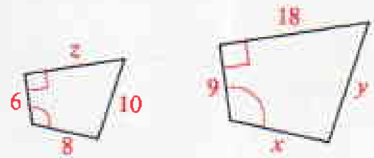
**7.** $\frac{a}{7} = \frac{b}{5}$ **8.** $7a = 5b$ **9.** $\frac{a+b}{b} = \frac{12}{7}$

**10.** If $\triangle ABC \sim \triangle RST$, $m\angle A = 45$, and $m\angle C = 60$, then $m\angle R =$ __?__, $m\angle T =$ __?__, and $m\angle S =$ __?__.

**The quadrilaterals shown are similar.**

**11.** The scale factor of the smaller quadrilateral to the larger quadrilateral is __?__.

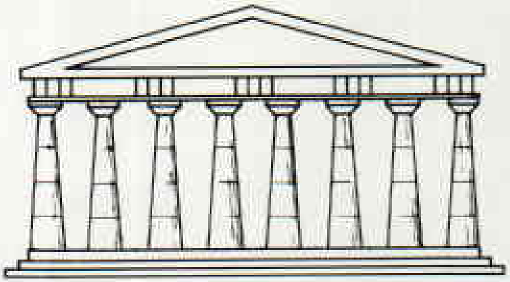
**12.** $x =$ __?__ **13.** $y =$ __?__ **14.** $z =$ __?__

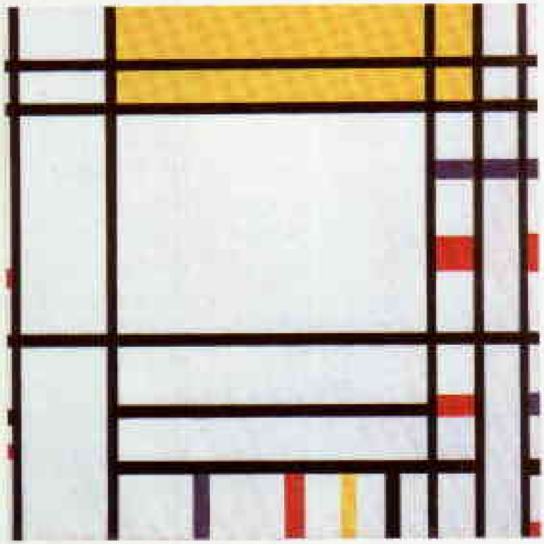



**15.** The measures of the angles of a hexagon are in the ratio 5:5:5:6:7:8. Find the measures.

## ♦ Calculator Key-In

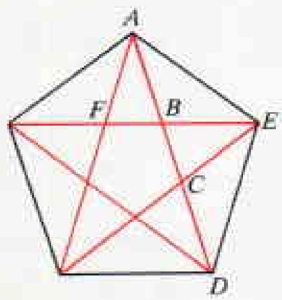
Before the pediment on top of the Parthenon in Athens was destroyed, the front of the building fit almost exactly into a golden rectangle. In a **golden rectangle,** the length $l$ and width $w$ satisfy the equation $\frac{l}{w} = \frac{l + w}{l}$. The ratio $\frac{l}{w}$ is called the **golden ratio.**





Over the centuries, artists and architects have found the golden rectangle to be especially pleasing to the eye. How many golden rectangles can you find in the painting by Piet Mondrian (1872–1944) that is shown?

## Exercises




**1.** A regular pentagon is shown. It happens to be true that $\frac{AD}{AC}$, $\frac{AC}{AB}$, and $\frac{AB}{BC}$ all equal the golden ratio. Measure the appropriate lengths to the nearest millimeter and compute the ratios with a calculator.

**2.** From the equation $\frac{l}{w} = \frac{l + w}{l}$ it can be shown that the numerical value of $\frac{l}{w}$ is $\frac{1 + \sqrt{5}}{2}$. Express the value of $\frac{l}{w}$, the golden ratio, as a decimal.

## Explorations

**These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.**

Draw any triangle and a median of the triangle. Measure and record the lengths of the sides and the median, the measures of the angles, and the perimeter of the triangle.

Change the scale of your triangle. Remeasure and record the lengths of the sides and the median, the measures of the angles, and the perimeter of the triangle.

Compare the measurements of the corresponding angles of the two triangles. What do you notice?

Divide the length of each side of the original triangle by the length of the corresponding side of the second triangle. What do you notice?

What do you know about the two triangles?

Divide the length of the median of the original triangle by the length of the median of the second triangle. What do you notice?

Divide the perimeter of the original triangle by the perimeter of the second triangle. What do you notice?

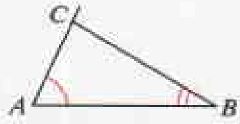
# Working with Similar Triangles

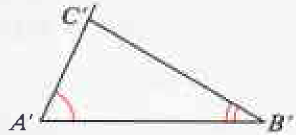
### Objectives

1. Use the AA Similarity Postulate, the SAS Similarity Theorem, and the SSS Similarity Theorem to prove triangles similar.
2. Use similar triangles to deduce information about segments or angles.
3. Apply the Triangle Proportionality Theorem and its corollary.
4. State and apply the Triangle Angle-Bisector Theorem.

## 7-4 *A Postulate for Similar Triangles*

You can always prove that two triangles are similar by showing that they satisfy the definition of similar polygons. However, there are simpler methods. The following experiment suggests the first of these methods: Two triangles are similar whenever two pairs of angles are congruent.




1. Draw any two segments $\overline{AB}$ and $\overline{A'B'}$.
2. Draw any angle at $A$ and a congruent angle at $A'$. Draw any angle at $B$ and a congruent angle at $B'$. Label points $C$ and $C'$ as shown. $\angle ACB \cong \angle A'C'B'$. (Why?)
3. Measure each pair of corresponding sides and compute an approximate decimal value for the ratio of their lengths:

$$\frac{AB}{A'B'} \qquad \frac{BC}{B'C'} \qquad \frac{AC}{A'C'}$$

4. Are the ratios computed in Step 3 approximately the same?

If you worked carefully, your answer in Step 4 was *yes*. Corresponding angles of the two triangles are congruent and corresponding sides are in proportion. By the definition of similar polygons, $\triangle ABC \sim \triangle A'B'C'$.

Whenever you draw two triangles with two angles of one triangle congruent to two angles of the other, you will find that the third angles are also congruent and that corresponding sides are in proportion.

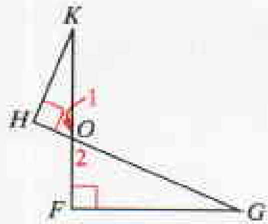
## Postulate 15 *AA Similarity Postulate*

**If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.**

**Example**

Given: $\angle H$ and $\angle F$ are rt. $\angle$s.

Prove: $HK \cdot GO = FG \cdot KO$




**Plan for Proof:** You can prove that $HK \cdot GO = FG \cdot KO$ if you show that $\frac{HK}{FG} = \frac{KO}{GO}$. You can get this proportion if you show that $\triangle HKO \sim \triangle FGO$.

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\angle 1 \cong \angle 2$ | 1. Vertical $\angle$s are $\cong$. |
| 2. $\angle H$ and $\angle F$ are rt. $\angle$s. | 2. Given |
| 3. $m\angle H = 90 = m\angle F$ | 3. Def. of a rt. $\angle$ |
| 4. $\angle H \cong \angle F$ | 4. Def. of $\cong$ $\angle$s |
| 5. $\triangle HKO \sim \triangle FGO$ | 5. AA Similarity Postulate |
| 6. $\frac{HK}{FG} = \frac{KO}{GO}$ | 6. Corr. sides of $\sim$ $\triangle$s are in proportion. |
| 7. $HK \cdot GO = FG \cdot KO$ | 7. A property of proportions |

The example shows one way to prove that the product of the lengths of two segments is equal to the product of the lengths of two other segments. You prove that two triangles are similar, write a proportion, and then apply the means-extremes property of proportions.

## Classroom Exercises

**In Exercises 1–8 $\triangle ABC \sim \triangle DEF$. Tell whether each statement must be true.**

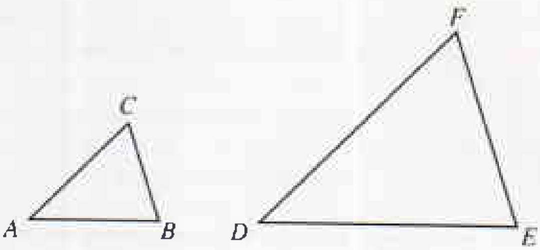
**1.** $\triangle BAC \sim \triangle EFD$

**2.** If $m\angle D = 45$, then $m\angle A = 45$.

**3.** If $m\angle B = 70$, then $m\angle F = 70$.

**4.** $AB:DE = EF:BC$

**5.** $AC:DF = AB:DE$




**6.** If $\frac{DF}{AC} = \frac{8}{5}$, then $\frac{m\angle D}{m\angle A} = \frac{8}{5}$.

**7.** If $\frac{DF}{AC} = \frac{8}{5}$, then $\frac{EF}{BC} = \frac{8}{5}$.

**8.** If the scale factor of $\triangle ABC$ to $\triangle DEF$ is 5 to 8, then the scale factor of $\triangle DEF$ to $\triangle ABC$ is 8 to 5.

**9.** One right triangle has an angle with measure 37. Another right triangle has an angle with measure 53. Are the two triangles similar? Explain.

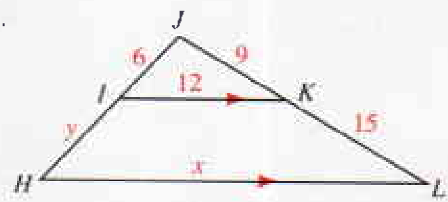
**10.** Name all pairs of congruent angles in the figure.

**11.** Complete.

**a.** $\triangle IKJ \sim$ _?_

**b.** $\frac{?}{x} = \frac{9}{24}$ and $x =$ _?_

**c.** $\frac{9}{24} = \frac{6}{?}$ and $y =$ _?_




**Exs. 10, 11**

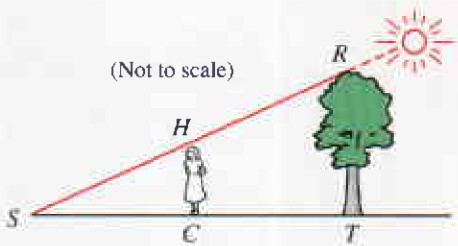
**12.** Suppose you want to show $AB \cdot YZ = CD \cdot WX$. What are some proportions that are equivalent to that equation?

**13.** Cecelia wanted to find the height of a certain tree for a report in her biology class. Her method used shadows as shown in the diagram. She measured the shadow of the tree and found it was 5 m long. She measured her shadow and found it was 0.8 m long.

**a.** $\triangle$_?_ $\sim \triangle$_?_

**b.** Complete: $\frac{SC}{?} = \frac{CH}{?}$

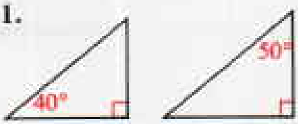
**c.** If Cecelia is 1.6 m tall, about how tall is the tree?



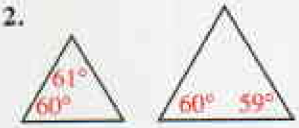
## Written Exercises

**Tell whether the triangles are similar or not similar. If you can't reach a conclusion, write *no conclusion is possible.***

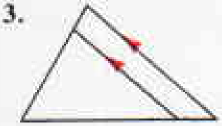
**A** **1.**



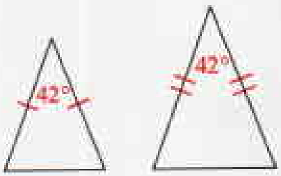

**2.**




**3.**

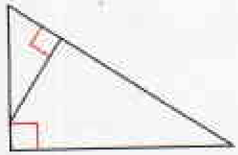


**4.**

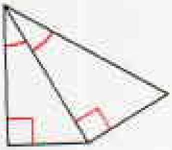



**5.**

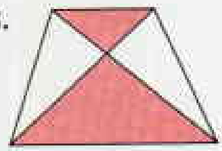

**6.**



**7.**

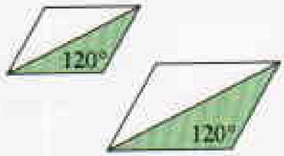


**8.**



Trapezoid given

**9.**




Parallelograms given

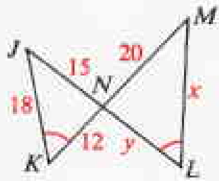
**10.** Complete.

**a.** $\triangle JKN \sim$ __?__

**b.** $\frac{JK}{?} = \frac{JN}{?} = \frac{KN}{?}$

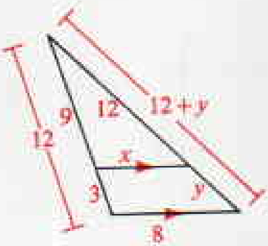
**c.** $\frac{15}{?} = \frac{18}{?}$ and $\frac{15}{?} = \frac{12}{?}$

**d.** $x =$ __?__ and $y =$ __?__

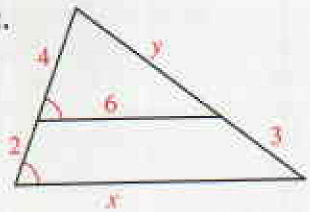



**Find the values of *x* and *y*.**

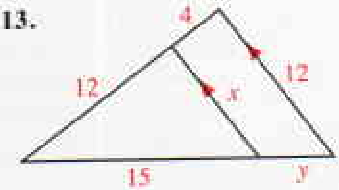
**11.**




**12.**

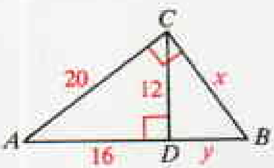



**13.**

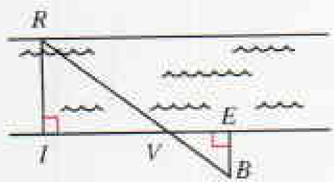



**B** **14. a.** Name two triangles that are similar to $\triangle ABC$.

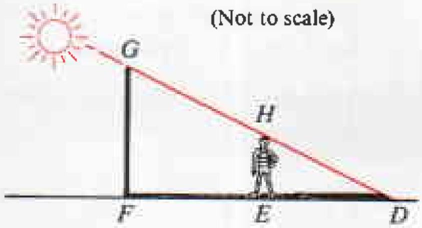
**b.** Find the values of $x$ and $y$.



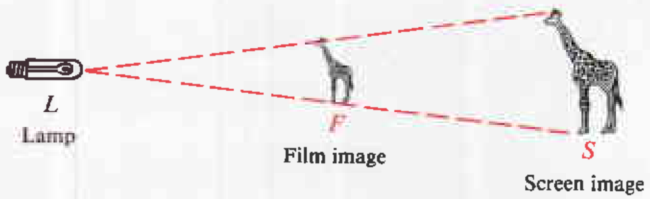

**15.** If $IV = 36$ m, $VE = 20$ m, and $EB = 15$ m, find the width, $RI$, of the river.



**16.** To estimate the height of a pole, a basketball player exactly 2 m tall stood so that the ends of his shadow and the shadow of the pole coincided. He found that $\overline{DE}$ and $\overline{DF}$ measured 1.6 m and 4.4 m, respectively. About how tall was the pole?

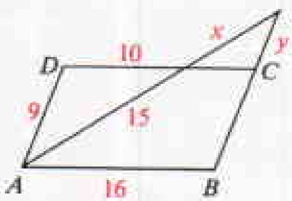



**17.** The diagram, *not* drawn to scale, shows a film being projected on a screen. $LF$ = 6 cm and $LS$ = 24 m. The screen image is 2.2 m tall. How tall is the film image?

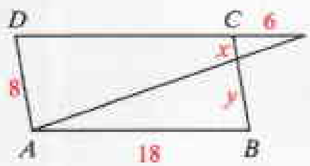



**In Exercises 18 and 19 *ABCD* is a parallelogram. Find the values of *x* and *y*.**

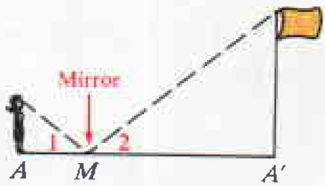
**18.**




**19.**



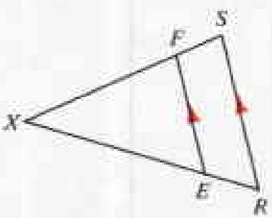

**20.** You can estimate the height of a flagpole by placing a mirror on level ground so that you see the top of the flagpole in it. The girl shown is 172 cm tall. Her eyes are about 12 cm from the top of her head. By measurement, $AM$ is about 120 cm and $A'M$ is about 4.5 m. From physics it is known that $\angle 1 \cong \angle 2$. Explain why the triangles are similar and find the approximate height of the pole.




**21.** Given: $\overline{EF} \parallel \overline{RS}$

Prove: **a.** $\triangle FXE \sim \triangle SXR$

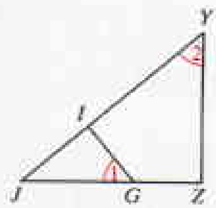
**b.** $\dfrac{FX}{SX} = \dfrac{EF}{RS}$




**22.** Given: $\angle 1 \cong \angle 2$

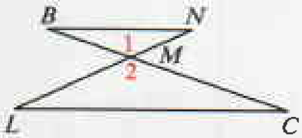
Prove: **a.** $\triangle JIG \sim \triangle JZY$

**b.** $\dfrac{JG}{JY} = \dfrac{GI}{YZ}$

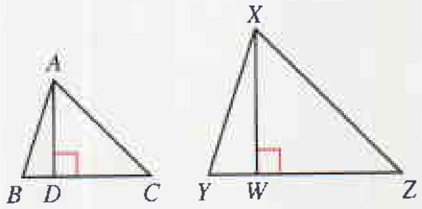


**23.** Given: $\angle B \cong \angle C$
Prove: $NM \cdot CM = LM \cdot BM$

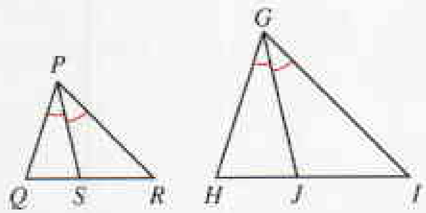
**24.** Given: $\overline{BN} \parallel \overline{LC}$
Prove: $BN \cdot LM = CL \cdot NM$



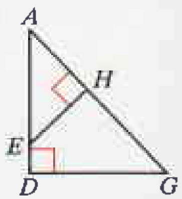

**25.** Given: $\triangle ABC \sim \triangle XYZ$; $\overline{AD}$ and $\overline{XW}$ are altitudes.
Prove: $\frac{AD}{XW} = \frac{AB}{XY}$



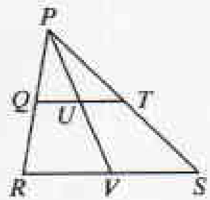

**26.** Given: $\triangle PQR \sim \triangle GHI$; $\overrightarrow{PS}$ and $\overrightarrow{GJ}$ are angle bisectors.
Prove: $\frac{PS}{GJ} = \frac{PQ}{GH}$



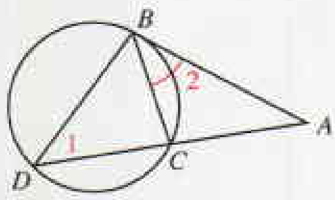

**27.** Given: $\overline{AH} \perp \overline{EH}$; $\overline{AD} \perp \overline{DG}$
Prove: $AE \cdot DG = AG \cdot HE$




**28.** Given: $\overline{QT} \parallel \overline{RS}$
Prove: $\frac{QU}{RV} = \frac{UT}{VS}$



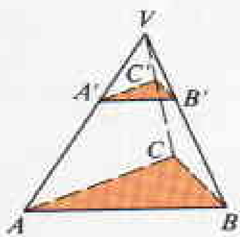

**29.** Given: $\angle 1 \cong \angle 2$
Prove: $(AB)^2 = AD \cdot AC$



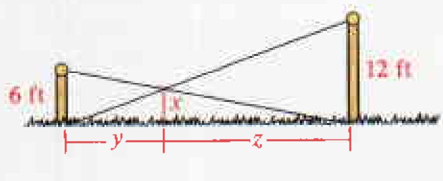

**In the diagram for Exercises 30 and 31, the plane of $\triangle A'B'C'$ is parallel to the plane of $\triangle ABC$.**

**30.** $VA' = 15$ and $A'A = 20$
**a.** If $VC' = 18$, then $VC =$ __?__.
**b.** If $VB = 49$, then $BB' =$ __?__.
**c.** If $A'B' = 24$, then $AB =$ __?__.

**31.** If $VA' = 10$, $VA = 25$, $AB = 20$, $BC = 14$, and $AC = 16$, find the perimeter of $\triangle A'B'C'$.




**C** **32.** Two vertical poles have heights 6 ft and 12 ft. A rope is stretched from the top of each pole to the bottom of the other. How far above the ground do the ropes cross? (*Hint*: The lengths $y$ and $z$ do not affect the answer.)



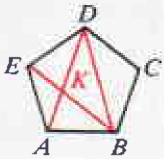
**In Exercises 33 and 34 write a paragraph proof for anything you are asked to prove.**

**33.** Given: Regular pentagon $ABCDE$

**a.** Make a large copy of the diagram.

**b.** Write the angle measures on your diagram.

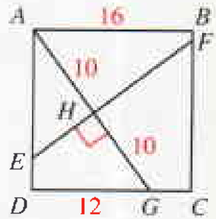
**c.** Prove that $\frac{DA}{DK} = \frac{DK}{AK}$.



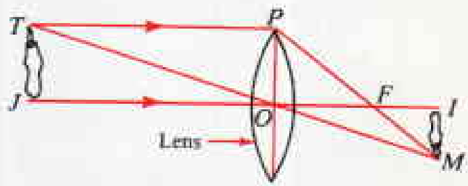

★ **34.** $ABCD$ is a square.

**a.** Find the distance from $H$ to each side of the square.

**b.** Find $BF$, $FC$, $CG$, $DE$, $EA$, $EH$, and $HF$.




★ **35.** Related to any doubly convex lens there is a focal distance $OF$. Physicists have determined experimentally that a vertical lens, a vertical object $\overline{JT}$ (with $\overline{JO}$ horizontal), a vertical image $\overline{IM}$, and a focus $F$ are related as shown in the diagram. Once the relationship is known, geometry can be used to establish a lens law:




$$\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{\text{focal distance}}$$

**a.** Prove that $\frac{1}{OJ} + \frac{1}{OI} = \frac{1}{OF}$.

**b.** Show algebraically that $OF = \frac{OJ \cdot OI}{OJ + OI}$.

## Challenges

**1.** Explain how to pass a plane through a cube so that the intersection is:

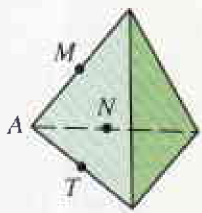
**a.** an equilateral triangle

**b.** a trapezoid

**c.** a pentagon

**d.** a hexagon

**2.** The six edges of the three-dimensional figure are congruent. Each of the four corners is cut off by a plane that passes through the midpoints of the three edges that intersect at that corner. For example, corner A is cut off by plane $MNT$. Describe the three-dimensional figure that remains.



## ♦ Computer Key-In

The sequence 1, 1, 2, 3, 5, 8, 13, 21, . . . is called a *Fibonacci sequence* after its discoverer, Leonardo Fibonacci, a thirteenth century mathematician. The first two terms are 1 and 1. You then add two consecutive terms to get the next term.

$$\begin{matrix}\text{1st}\\ \text{term}\end{matrix} + \begin{matrix}\text{2nd}\\ \text{term}\end{matrix} = \begin{matrix}\text{3rd}\\ \text{term}\end{matrix} \qquad \begin{matrix}\text{2nd}\\ \text{term}\end{matrix} + \begin{matrix}\text{3rd}\\ \text{term}\end{matrix} = \begin{matrix}\text{4th}\\ \text{term}\end{matrix} \qquad \begin{matrix}\text{3rd}\\ \text{term}\end{matrix} + \begin{matrix}\text{4th}\\ \text{term}\end{matrix} = \begin{matrix}\text{5th}\\ \text{term}\end{matrix}$$

The following computer program computes the first twenty-five terms of the Fibonacci sequence shown above and finds the ratio of any term to its preceding term. For example, we want to look at the ratios

$$\frac{1}{1} = 1,\ \frac{2}{1} = 2,\ \frac{3}{2} = 1.5,\ \frac{5}{3} \approx 1.66667, \text{ and so on.}$$

```
10  PRINT "TERM NO.", "TERM", "RATIO"
20  LET A = 1
30  LET B = 1
40  PRINT "1",A,"-"
50  FOR N = 2 TO 25
60    LET D = B/A
70    LET E = 10000 * D
80    LET F = INT(E)
90    LET G = F/10000
100   PRINT N,B,G
110   LET C = B + A
120   LET A = B
130   LET B = C
140 NEXT N
150 END
```

## Exercises

**Type the program into your computer and use it in Exercises 1–4.**

**1.** RUN the given computer program. As the terms become larger, what happens to the values of the ratios?

**2.** Suppose another sequence is formed by choosing starting numbers different from 1 and 1. For example, suppose the sequence is 3, 11, 14, 25, 39, . . . , where the pattern for creating the terms of the sequence is still the same. Change lines 20 and 30 to:

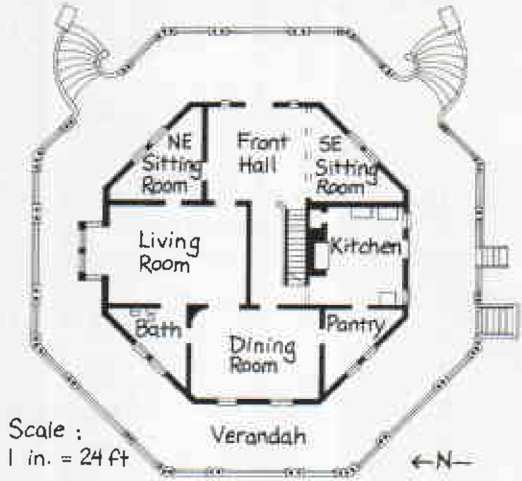
```
20  LET A = 3
30  LET B = 11
```

RUN the modified program. What happens to the values of the ratios as the terms become larger and larger?

**3.** Modify the program again so that another pair of starting numbers is used and the first thirty terms are computed. RUN the program. What can you conclude from the results?

**4.** Compare this ratio to the golden ratio calculated in Exercise 2 on page 253. Do you see a connection?

## Application — *Scale Drawings*




This "octagon house" was built in Irvington, New York, in 1860. The plan shows the rooms on the first floor. The scale on this *scale drawing* tells you that a length of 1 in. on the plan represents a true length of 24 ft.

$$\frac{\text{Plan length in inches}}{\text{True length in feet}} = \frac{1}{24}$$

The following examples show how you can use this formula to find actual dimensions of the house from the plan or to convert dimensions of full-sized objects to plan size.

The verandah measures $\frac{3}{8}$ in. wide on the plan. Find its true width, $T$.

$\frac{\frac{3}{8}}{T} = \frac{1}{24}$, so $1 \cdot T = \frac{3}{8} \cdot 24$

$T = 9$ The real verandah is 9 ft wide.

A sofa is 6 ft long. Find its plan length, $P$.

$\frac{P}{6} = \frac{1}{24}$, so $24 \cdot P = 6 \cdot 1$

$P = \frac{1}{4}$ The plan length is $\frac{1}{4}$ in.

## Exercises

1. Find the true length and width of the dining room.
2. A rug measures 9 ft by $7\frac{1}{2}$ ft. What would its dimensions be on the floor plan? Would it fit in the northeast sitting room?
3. If a new floor plan is drawn with a scale of 1 in. = 10 ft, how many times longer is each line segment on the new plan than the corresponding segment on the plan shown?
4. Suppose that on the architect's drawings each side of the verandah (the outer octagon) measured 12 in. What was the scale of these drawings?

# 7-5 *Theorems for Similar Triangles*

You can prove two triangles similar by using the definition of similar polygons or by using the AA Postulate. Of course, in practice you would always use the AA Postulate instead of the definition. (Why?) Two additional methods are established in the theorems below. The proofs involve proportions, congruence, and similarity.

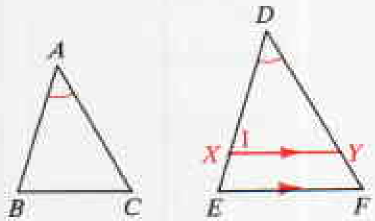
## Theorem 7-1 *SAS Similarity Theorem*

**If an angle of one triangle is congruent to an angle of another triangle and the sides including those angles are in proportion, then the triangles are similar.**

Given: $\angle A \cong \angle D$;

$\frac{AB}{DE} = \frac{AC}{DF}$

Prove: $\triangle ABC \sim \triangle DEF$



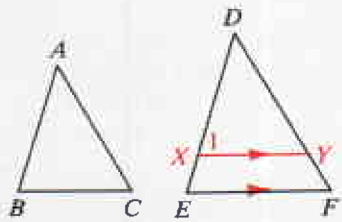

**Plan for Proof:** Take $X$ on $\overline{DE}$ so that $DX = AB$. Draw a line through $X$ parallel to $\overleftrightarrow{EF}$. Then $\triangle DEF \sim \triangle DXY$ and $\frac{DX}{DE} = \frac{DY}{DF}$. Since $\frac{AB}{DE} = \frac{AC}{DF}$ and $DX = AB$, you can deduce that $DY = AC$. Thus $\triangle ABC \cong \triangle DXY$ by SAS. Therefore, $\triangle ABC \sim \triangle DXY$, and $\triangle ABC \sim \triangle DEF$ by the Transitive Property of Similarity.

## Theorem 7-2 *SSS Similarity Theorem*

**If the sides of two triangles are in proportion, then the triangles are similar.**

Given: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

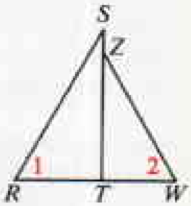
Prove: $\triangle ABC \sim \triangle DEF$




**Plan for Proof:** Take $X$ on $\overline{DE}$ so that $DX = AB$. Draw a line through $X$ parallel to $\overleftrightarrow{EF}$. Then $\triangle DEF \sim \triangle DXY$ and $\frac{DX}{DE} = \frac{XY}{EF} = \frac{DY}{DF}$. With the given proportion and $DX = AB$, you can deduce that $AC = DY$ and $BC = XY$. Thus $\triangle ABC \cong \triangle DXY$ by SSS. Therefore, $\triangle ABC \sim \triangle DXY$, and $\triangle ABC \sim \triangle DEF$ by the Transitive Property of Similarity.

To prove two polygons similar, you might need to compare the corresponding sides. As shown in the example below, a useful technique is to compare the longest sides, the shortest sides, and so on.

**Example** Can the information given in each part be used to prove $\triangle RST \sim \triangle WZT$? If so, how?




**a.** $RS = 18$, $ST = 15$, $RT = 10$, $WT = 6$, $ZT = 9$, $WZ = 10.8$

**b.** $\angle 1 \cong \angle 2$, $\frac{WZ}{RS} = \frac{TZ}{TS}$

**c.** $\overline{ST} \perp \overline{RW}$, $ST = 32$, $SZ = 8$, $RT = 20$, $WT = 15$

**Solution**

**a.** Comparing the longest sides, $\frac{RS}{WZ} = \frac{18}{10.8} = \frac{5}{3}$.

Comparing the shortest sides, $\frac{RT}{WT} = \frac{10}{6} = \frac{5}{3}$.

Comparing the remaining sides, $\frac{ST}{ZT} = \frac{15}{9} = \frac{5}{3}$.

Thus, $\frac{RS}{WZ} = \frac{RT}{WT} = \frac{ST}{ZT}$.

$\triangle RST \sim \triangle WZT$ by the SSS Similarity Theorem.

**b.** Notice that $\angle 1$ and $\angle 2$ are not the angles included by the sides that are in proportion. Therefore, the triangles cannot be proved similar.

**c.** Comparing the shorter legs, $\frac{RT}{WT} = \frac{20}{15} = \frac{4}{3}$.

Comparing the other legs, $\frac{ST}{ZT} = \frac{32}{32 - 8} = \frac{32}{24} = \frac{4}{3}$.

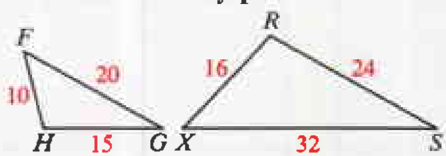
$\frac{RT}{WT} = \frac{ST}{ZT}$ and $\angle RTS \cong \angle WTS$.

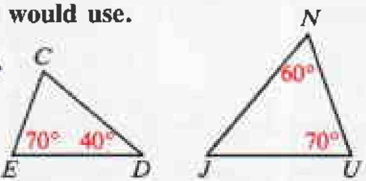
$\triangle RST \sim \triangle WZT$ by the SAS Similarity Theorem.

The perimeters of similar polygons are in the same ratio as the corresponding sides. By using similar triangles, you can prove that corresponding segments such as diagonals of similar polygons also have this ratio. (See Exercises 25 and 26, page 259 and Exercises 17 and 18, page 267.)

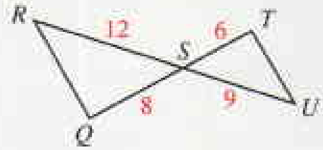
## Classroom Exercises

**Can the two triangles shown be proved similar? If so, state the similarity and tell which similarity postulate or theorem you would use.**

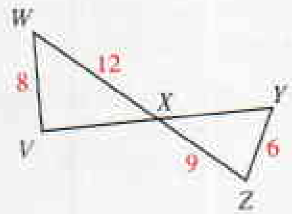
1. 


2. 

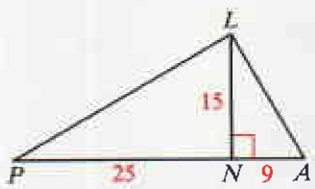
**3.**



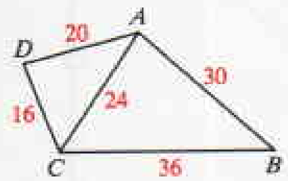

**4.**




**5.**



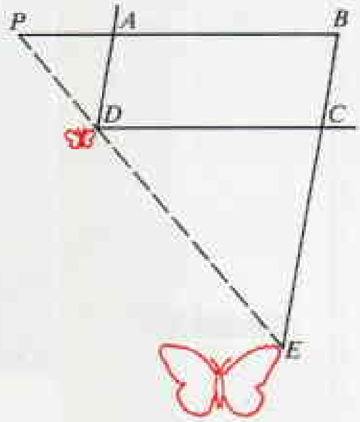

**6.**

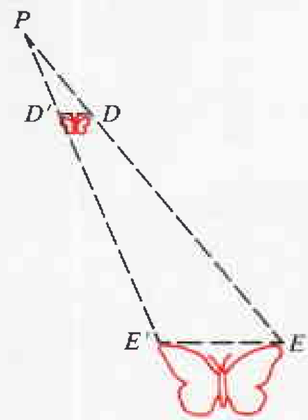



**7.** Suppose you want to prove that $\triangle RST \sim \triangle XYZ$ by the SSS Similarity Theorem. State the extended proportion you would need to prove first.

**8.** Suppose you want to prove that $\triangle RST \sim \triangle XYZ$ by the SAS Similarity Theorem. If you know that $\angle R \cong \angle X$, what else do you need to prove?

**9.** A *pantograph* is a tool for enlarging or reducing maps and drawings. Four bars are pinned together at $A$, $B$, $C$, and $D$ so that $ABCD$ is a parallelogram and points $P$, $D$, and $E$ lie on a line. Point $P$ is fixed to the drawing board. To enlarge a figure, the artist inserts a stylus at $D$, a pen or pencil at $E$, and guides the stylus so that it traces the original. As $D$ moves, the angles of the parallelogram change, but $P$, $D$, and $E$ remain collinear. Suppose $PA$ is 3 units and $AB$ is 7 units.




**a.** Explain why $\triangle PBE \sim \triangle PAD$.

**b.** What is the ratio of $PB$ to $PA$?

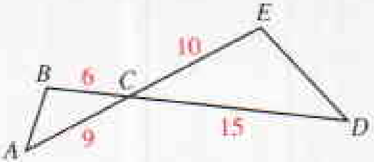
**c.** What is the ratio of $PE$ to $PD$?

**d.** What is the ratio of the butterfly's wingspan, $E'E$, in the enlargement to its wingspan, $D'D$, in the original?

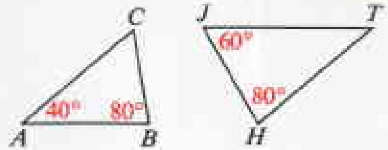
## Written Exercises

**Name two similar triangles. What postulate or theorem justifies your answer?**

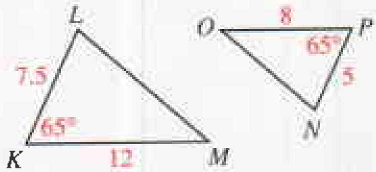
**A** **1.**



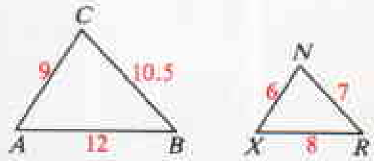

**2.**



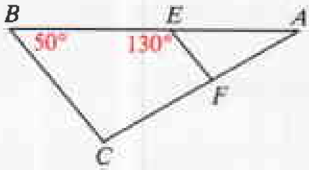

**3.**



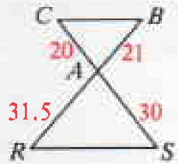

**4.**




**5.**




**6.**

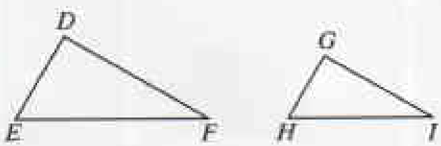



**One triangle has vertices $A$, $B$, and $C$. Another has vertices $T$, $R$, and $I$. Are the two triangles similar? If so, state the similarity and the scale factor.**

| | $AB$ | $BC$ | $AC$ | $TR$ | $RI$ | $TI$ |
|---|---|---|---|---|---|---|
| **7.** | 6 | 8 | 10 | 9 | 12 | 15 |
| **8.** | 6 | 8 | 10 | 12 | 22 | 16 |
| **9.** | 6 | 8 | 10 | 20 | 25 | 15 |
| **10.** | 6 | 8 | 10 | 10 | 7.5 | 12.5 |

**11.** Given: $\frac{DE}{GH} = \frac{DF}{GI} = \frac{EF}{HI}$

Prove: $\angle E \cong \angle H$

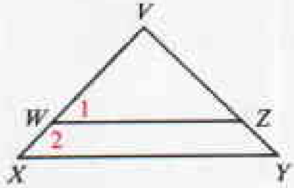



**12.** Given: $\frac{DE}{GH} = \frac{EF}{HI}$; $\angle E \cong \angle H$

Prove: $\frac{EF}{HI} = \frac{DF}{GI}$

**B** **13.** Given: $\frac{VW}{VX} = \frac{VZ}{VY}$

Prove: $\overline{WZ} \parallel \overline{XY}$



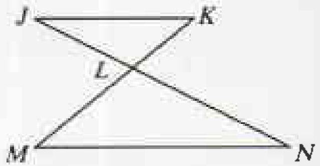

**14.** Given: $\frac{VW}{VY} = \frac{VZ}{VX}$

Which one(s) of the following *must* be true?

(1) $\triangle VWZ \sim \triangle VXY$ (2) $\overline{WZ} \parallel \overline{XY}$ (3) $\angle 1 \cong \angle Y$

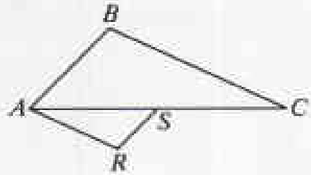
**15.** Given: $\frac{JL}{NL} = \frac{KL}{ML}$

Prove: $\angle J \cong \angle N$




**16.** Given: $\frac{AB}{SR} = \frac{BC}{RA} = \frac{CA}{AS}$

Prove: $\overline{BC} \parallel \overline{AR}$



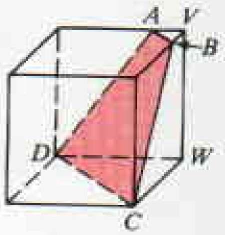

**Draw and label a diagram. List, in terms of the diagram, what is given and what is to be proved. Then write a proof.**

**17.** If two triangles are similar, then the lengths of corresponding medians are in the same ratio as the lengths of corresponding sides.

**18.** If two quadrilaterals are similar, then the lengths of corresponding diagonals are in the same ratio as the lengths of corresponding sides.

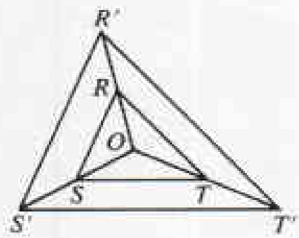
**19.** If the vertex angle of one isosceles triangle is congruent to the vertex angle of another isosceles triangle, then the triangles are similar.

**20.** The faces of a cube are congruent squares. The cube shown is cut by plane $ABCD$. $VA = VB$ and $VW = 4 \cdot VA$. Find, in terms of $AB$, the length of the median of trap. $ABCD$.




**21.** Given: $OR' = 2 \cdot OR$; $OS' = 2 \cdot OS$; $OT' = 2 \cdot OT$

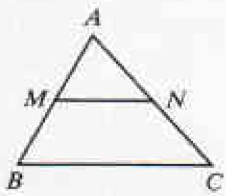
Prove: $\triangle RST \sim \triangle R'S'T'$




**22.** Prove Theorem 5-11 on page 178: The segment that joins the midpoints of two sides of a triangle is parallel to the third side and is half as long as the third side.

Given: $M$ is the midpoint of $\overline{AB}$;
$N$ is the midpoint of $\overline{AC}$.

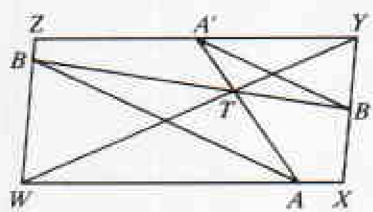
Prove: $\overline{MN} \parallel \overline{BC}$; $MN = \frac{1}{2}BC$




**C** **23.** Given: ▱ $WXYZ$

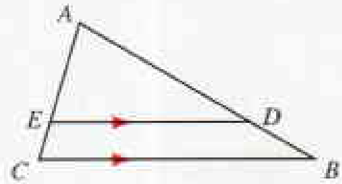
Prove: $\triangle ATB \sim \triangle A'TB'$

(*Hint:* Show that $\frac{AT}{A'T}$ and $\frac{BT}{B'T}$ both equal $\frac{TW}{TY}$.)



## Explorations

**These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.**




1. Draw any triangle $ABC$. Choose a point $D$ on $\overline{AB}$. Draw a line through $D$ parallel to $\overline{BC}$ and intersecting $\overline{AC}$ at point $E$.

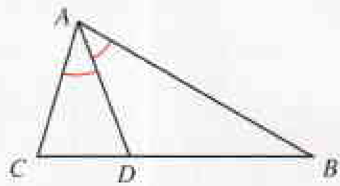
   What do you know about $\triangle ABC$ and $\triangle ADE$?

   What do you know about $\frac{AE}{AC}$ and $\frac{AD}{AB}$?

   Calculate $\frac{AE}{EC}$ and $\frac{AD}{DB}$. What do you notice?

   Calculate $\frac{EC}{AC}$ and $\frac{DB}{AB}$. What do you notice?

   Repeat on other triangles. What do you notice?




2. Draw any triangle $ABC$. Draw the bisector of $\angle A$ intersecting $\overline{CB}$ at point $D$.

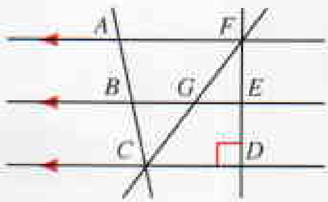
   Measure and record the four lengths: $AB$, $AC$, $BD$, $DC$.

   Calculate $\frac{BD}{DC}$ and $\frac{AB}{AC}$. What do you notice?

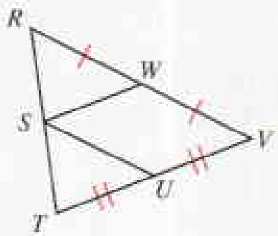
   Repeat on other triangles. What do you notice?

## Mixed Review Exercises

**Complete.**




1. Given: $\overline{AF} \parallel \overline{BE} \parallel \overline{CD}$; $\overline{AB} \cong \overline{BC}$
   a. $\overline{GF} \cong$ _?_ and $\overline{ED} \cong$ _?_
   b. If $AB = 9$, then $AC =$ _?_.
   c. If $FG = 3x + 2$ and $GC = 7x - 10$, then $x =$ _?_.
   d. $m\angle FEG =$ _?_




2. Given: $W$ and $U$ are the midpoints of $\overline{RV}$ and $\overline{VT}$; $\overline{SW} \parallel \overline{VT}$
   a. $S$ is the _?_ of $\overline{RT}$ and $\overline{SU} \parallel$ _?_.
   b. If $TV = 12x + 4$ and $SW = 3x + 8$, then $x =$ _?_.
   c. If $RW = 4y + 1$ and $SU = 9y - 19$, then $y =$ _?_.

# 7-6 *Proportional Lengths*

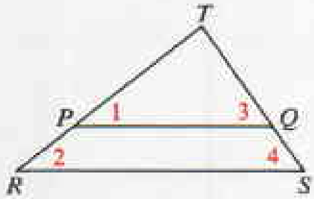
Points $L$ and $M$ lie on $\overline{AB}$ and $\overline{CD}$, respectively. If $\frac{AL}{LB} = \frac{CM}{MD}$, we say that $\overline{AB}$ and $\overline{CD}$ are **divided proportionally.**


## Theorem 7-3 *Triangle Proportionality Theorem*

**If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.**

Given: $\triangle RST$; $\overleftrightarrow{PQ} \parallel \overline{RS}$

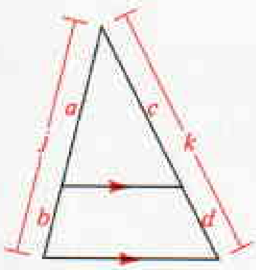
Prove: $\frac{RP}{PT} = \frac{SQ}{QT}$




**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\overleftrightarrow{PQ} \parallel \overline{RS}$ | 1. ___?___ |
| 2. $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$ | 2. ___?___ |
| 3. $\triangle RST \sim \triangle PQT$ | 3. ___?___ |
| 4. $\frac{RT}{PT} = \frac{ST}{QT}$ | 4. Corr. sides of ~ △ are in proportion. |
| 5. $RT = RP + PT$; $ST = SQ + QT$ | 5. ___?___ |
| 6. $\frac{RP + PT}{PT} = \frac{SQ + QT}{QT}$ | 6. ___?___ |
| 7. $\frac{RP}{PT} = \frac{SQ}{QT}$ | 7. A property of proportions (Property 1(d), page 245.) |

We will use the Triangle Proportionality Theorem to justify any proportion equivalent to $\frac{RP}{PT} = \frac{SQ}{QT}$. For the diagram at the right, some of the proportions that may be justified by the Triangle Proportionality Theorem include:

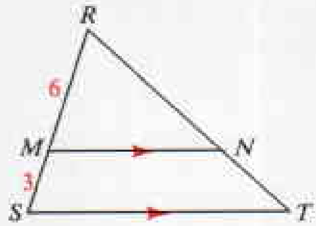



$$\frac{a}{j} = \frac{c}{k} \qquad \frac{a}{c} = \frac{j}{k} \qquad \frac{b}{j} = \frac{d}{k}$$

$$\frac{a}{b} = \frac{c}{d} \qquad \frac{a}{c} = \frac{b}{d} \qquad \frac{b}{d} = \frac{j}{k}$$

**Example** Find the numerical value.

**a.** $\frac{TN}{NR}$ **b.** $\frac{TR}{NR}$ **c.** $\frac{RN}{RT}$




**Solution**

**a.** $\frac{TN}{NR} = \frac{SM}{MR} = \frac{3}{6} = \frac{1}{2}$

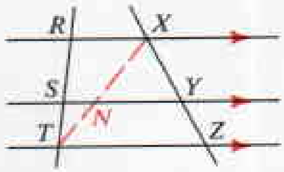
**b.** $\frac{TR}{NR} = \frac{SR}{MR} = \frac{9}{6} = \frac{3}{2}$

**c.** $\frac{RN}{RT} = \frac{RM}{RS} = \frac{6}{9} = \frac{2}{3}$

Compare the following corollary with Theorem 5-9 on page 177.

## Corollary

**If three parallel lines intersect two transversals, then they divide the transversals proportionally.**



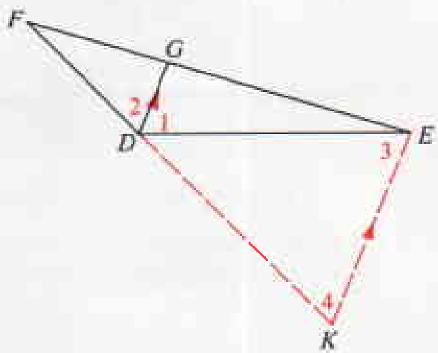

Given: $\overleftrightarrow{RX} \parallel \overleftrightarrow{SY} \parallel \overleftrightarrow{TZ}$

Prove: $\frac{RS}{ST} = \frac{XY}{YZ}$

**Plan for Proof:** Draw $\overline{TX}$, intersecting $\overleftrightarrow{SY}$ at $N$. Note that $\overleftrightarrow{SY}$ is parallel to one side of $\triangle RTX$, and also to one side of $\triangle TXZ$. You can apply the Triangle Proportionality Theorem to both of these triangles. Use those proportions to show $\frac{RS}{ST} = \frac{XY}{YZ}$.

## Theorem 7-4 *Triangle Angle-Bisector Theorem*

**If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.**



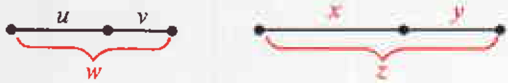

Given: $\triangle DEF$; $\overrightarrow{DG}$ bisects $\angle FDE$.

Prove: $\frac{GF}{GE} = \frac{DF}{DE}$

**Plan for Proof:** Draw a line through $E$ parallel to $\overrightarrow{DG}$ and intersecting $\overrightarrow{FD}$ at $K$. Apply the Triangle Proportionality Theorem to $\triangle FKE$. $\triangle DEK$ is isosceles with $DK = DE$. Substitute this into your proportion to complete the proof.

## Classroom Exercises

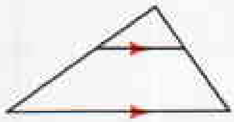
1. The two segments are divided proportionally. State several correct proportions.



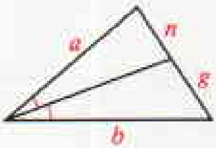

2. Complete the proportions stated informally below.

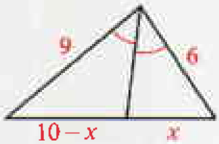
$\frac{\text{lower left}}{\text{whole left}} = \frac{\text{lower right}}{?}$ $\frac{\text{upper left}}{\text{lower left}} = \frac{?}{?}$

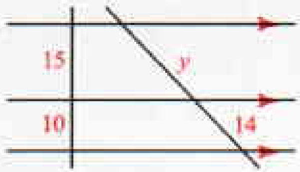
$\frac{\text{upper left}}{\text{whole left}} = \frac{\text{upper parallel}}{?} = \frac{?}{?}$



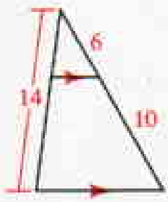
**State a proportion for each diagram.**

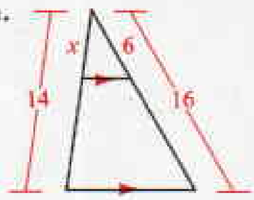
3. 

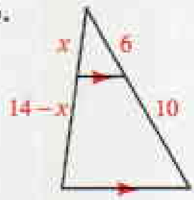

4. 

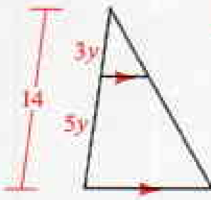

5. 


6. Suppose you want to find the length of the upper left segment in the diagram at the right. Three methods are suggested below. Complete each solution.



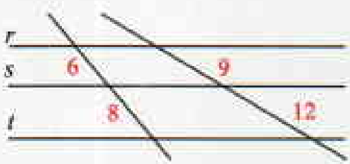

a. 


b. 

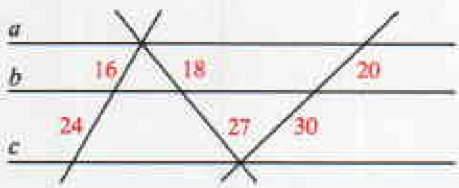

c. 


7. Explain why the expressions $3y$ and $5y$ can be used in Exercise 6(c).

8. The converse of the corollary of the Triangle Proportionality Theorem is: If three lines divide two transversals proportionally, then the lines are parallel. Is the converse true? (*Hint:* Can you draw a diagram with lengths like those shown below, but in which lines *r*, *s*, and *t* are not parallel?)




**Ex. 8**




**Ex. 9**

9. Must lines *a*, *b*, and *c* shown above be parallel? Explain.

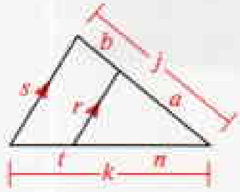
## Written Exercises

**A**

1. Tell whether the proportion is correct.

   a. $\frac{r}{s} = \frac{a}{b}$ b. $\frac{j}{a} = \frac{s}{r}$ c. $\frac{a}{b} = \frac{n}{t}$

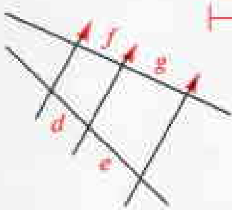
   d. $\frac{t}{k} = \frac{a}{j}$ e. $\frac{r}{s} = \frac{n}{k}$ f. $\frac{b}{j} = \frac{t}{k}$



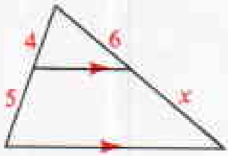

2. Tell whether the proportion is correct.

   a. $\frac{d}{f} = \frac{g}{e}$ b. $\frac{f}{g} = \frac{e}{d}$

   c. $\frac{g}{f} = \frac{e}{d}$ d. $\frac{d}{f} = \frac{e}{g}$

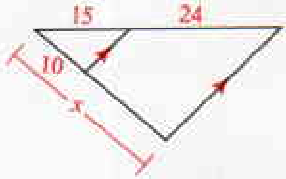


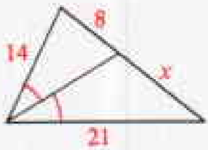

**Find the value of x.**

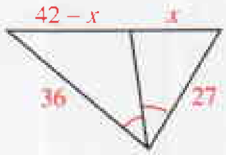
3.


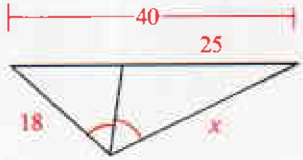

4.



5.


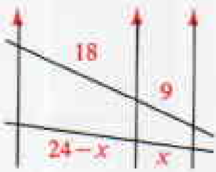

6.


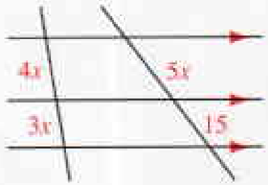

7.



8.



9.



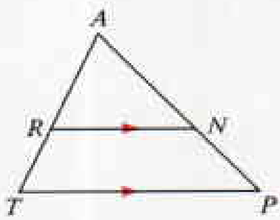
10.



11.



**Copy the table and fill in as many spaces *as possible*. It may help to draw a new sketch for each exercise and label lengths as you find them.**

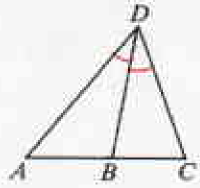
**B**

| | *AR* | *RT* | *AT* | *AN* | *NP* | *AP* | *RN* | *TP* |
|---|---|---|---|---|---|---|---|---|
| 12. | 6 | 4 | ? | 9 | ? | ? | ? | 15 |
| 13. | ? | ? | ? | ? | 6 | 16 | ? | ? |
| 14. | 18 | ? | ? | ? | ? | ? | 30 | 40 |
| 15. | 12 | ? | 20 | ? | ? | 30 | 15 | ? |
| 16. | ? | 18 | ? | ? | 26 | ? | 12 | 36 |
| 17. | ? | 8 | 16 | 6 | ? | ? | ? | ? |



**18.** Prove the corollary of the Triangle Proportionality Theorem.

**19.** Prove the Triangle Angle-Bisector Theorem.




**Complete.**

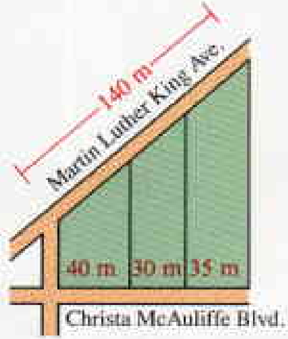
**20.** $AD = 21$, $DC = 14$, $AC = 25$, $AB = \underline{\ ?\ }$

**21.** $AC = 60$, $CD = 30$, $AD = 50$, $BC = \underline{\ ?\ }$

**22.** $AB = 27$, $BC = x$, $CD = \frac{4}{3}x$, $AD = x$, $AC = \underline{\ ?\ }$

**23.** $AB = 2x - 12$, $BC = x$, $CD = x + 5$, $AD = 2x - 4$, $AC = \underline{\ ?\ }$

**24.** Three lots with parallel side boundaries extend from the avenue to the boulevard as shown. Find, to the nearest tenth of a meter, the frontages of the lots on Martin Luther King Avenue.




**Ex. 24**

**25.** The lengths of the sides of $\triangle ABC$ are $BC = 12$, $CA = 13$, and $AB = 14$. If $M$ is the midpoint of $\overline{CA}$, and $P$ is the point where $\overline{CA}$ is cut by the bisector of $\angle B$, find $MP$.

**26.** Prove: If a line bisects both an angle of a triangle and the opposite side, then the triangle is isosceles.

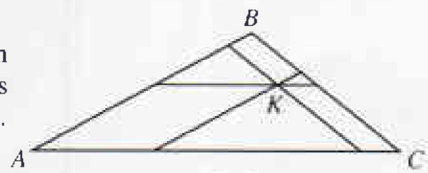
**C** **27.** Discover and prove a theorem about planes and transversals suggested by the corollary of the Triangle Proportionality Theorem.

**28.** Prove that there cannot be a triangle in which the trisectors of an angle also trisect the opposite side.

**29.** Can there exist a $\triangle ROS$ in which the trisectors of $\angle O$ intersect $\overline{RS}$ at $D$ and $E$, with $RD = 1$, $DE = 2$, and $ES = 4$? Explain.

**30.** Angle $E$ of $\triangle ZEN$ is obtuse. The bisector of $\angle E$ intersects $\overline{ZN}$ at $X$. $J$ and $K$ lie on $\overline{ZE}$ and $\overline{NE}$ with $ZJ = ZX$ and $NK = NX$. Discover and prove something about quadrilateral $ZNKJ$.

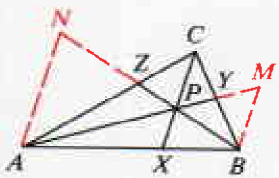
★ **31.** In $\triangle ABC$, $AB = 8$, $BC = 6$, and $AC = 12$. Each of the three segments drawn through point $K$ has length $x$ and is parallel to a side of the triangle. Find the value of $x$.




★ **32.** In $\triangle RST$, $U$ lies on $\overline{TS}$ with $TU:US = 2:3$. $M$ is the midpoint of $\overline{RU}$. $\overrightarrow{TM}$ intersects $\overline{RS}$ in $V$. Find the ratio $RV:RS$.

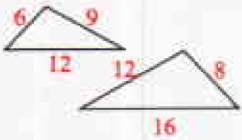
★ **33.** Prove *Ceva's Theorem:* If $P$ is any point inside $\triangle ABC$, then $\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZA} = 1$.

(*Hint:* Draw lines parallel to $\overline{CX}$ through $A$ and $B$. Apply the Triangle Proportionality Theorem to $\triangle ABM$. Show that $\triangle APN \sim \triangle MPB$, $\triangle BYM \sim \triangle CYP$, and $\triangle CZP \sim \triangle AZN$.)

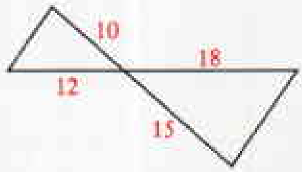


# Self-Test 2

**State the postulate or theorem you can use to prove that two triangles are similar.**

**1.** 


**2.** 

**3.** 

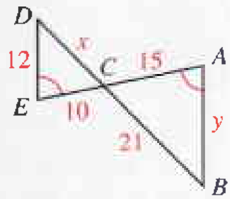

**4.** Complete.

**a.** $\triangle ABC \sim$ _?_

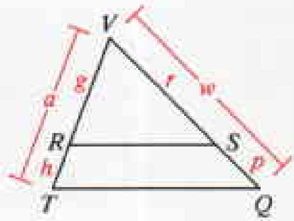
**b.** $\frac{AB}{?} = \frac{AC}{?} = \frac{BC}{?}$

**c.** $\frac{15}{?} = \frac{21}{?}$, and $x =$ _?_

**d.** $\frac{15}{?} = \frac{?}{12}$, and $y =$ _?_




**In the figure, it is given that $\overline{RS} \parallel \overline{TQ}$. Complete each proportion.**



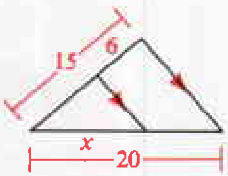

**5.** $\frac{g}{h} = \frac{?}{p}$

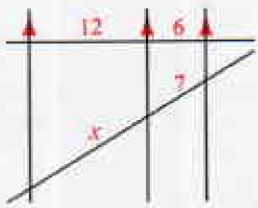
**6.** $\frac{a}{h} = \frac{w}{?}$

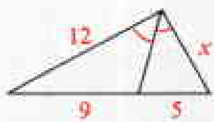
**7.** $\frac{r}{g} = \frac{p}{?}$

**8.** $\frac{h}{p} = \frac{?}{w}$

**Find the value of $x$.**

**9.** 

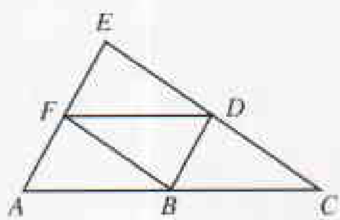

**10.** 


**11.** 


## Challenge

Given: $\overline{FD} \parallel \overline{AC}$; $\overline{BD} \parallel \overline{AE}$; $\overline{FB} \parallel \overline{EC}$

Show that $B$, $D$, and $F$ are midpoints of $\overline{AC}$, $\overline{CE}$, and $\overline{EA}$.



**Extra**

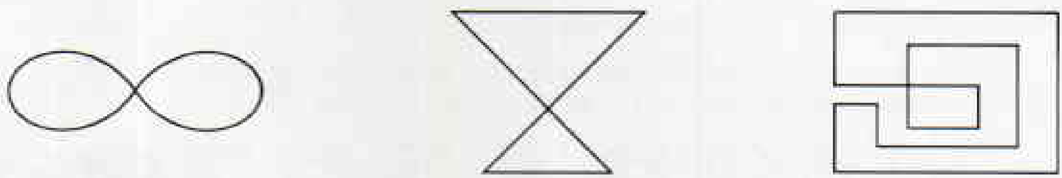
# Topology

In the geometry we have been studying, our interest has been in congruent figures and similar figures, that is, figures with the same size and shape or at least the same shape. If we were studying the branch of geometry called *topology*, we would be interested in properties of figures that are even more basic than size and shape. For example, imagine taking a rubber band and stretching it into all kinds of figures.

These figures have different sizes and shapes, but they still have something in common: Each one can be turned into any of the others by stretching and bending the rubber band. In topology figures are classified according to this kind of family resemblance. Figures that can be stretched, bent, or molded into the same shape without cutting or puncturing belong to the same family and are called *topologically equivalent*. Thus circles, squares, and triangles are equivalent. Likewise the straight line segment and wiggly curves below are equivalent.

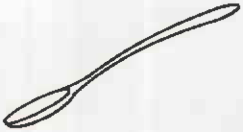
Notice that to make one of the figures above out of the rubber band you would have to cut the band, so these two-ended curves are not equivalent to the closed curves in the first illustration.

Suppose that in the plane figures below, the lines are joined where they cross. Then these figures belong to a third family. They are equivalent to each other but not to any of the figures above.

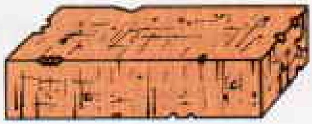


One of the goals of topology is to identify and describe the different families of equivalent figures. A person who studies topology (called a *topologist*) is interested in classifying solid figures as well as figures in a plane. For example, a topologist considers an orange, a teaspoon, and a brick equivalent to each other.

Orange



Teaspoon



Brick

In fact, a doughnut is topologically equivalent to a coffee cup. (See the diagrams below.) For this reason, a topologist has been humorously described as a mathematician who can't tell the difference between a doughnut and a coffee cup!

Think of the objects as made of modeling clay.

Push thumb into clay to make room for coffee.

## Exercises

**In each exercise tell which figure is *not* topologically equivalent to the rest. Exercises 1 and 2 show plane figures.**

**1. a.**  **b.**  **c.**  **d.** 

**2. a.**  **b.**  **c.**  **d.** 

**3. a.** solid ball **b.** hollow ball **c.** crayon **d.** comb

**4. a.** saucer **b.** house key **c.** coffee cup **d.** wedding ring

**5. a.** hammer **b.** screwdriver **c.** thimble **d.** sewing needle

**6.** Group the block numbers shown into three groups such that the numbers in each group are topologically equivalent to each other.


**7.** Make a series of drawings showing that the items in each pair are topologically equivalent to each other.

**a.** a drinking glass and a dollar bill **b.** a tack and a paper clip

# Chapter Summary

1. The ratio of $a$ to $b$ is the quotient $\frac{a}{b}$ ($b$ cannot be 0). The ratio $\frac{a}{b}$ can also be written $a:b$.
2. A proportion is an equation, such as $\frac{a}{b} = \frac{c}{d}$, stating that two ratios are equal.
3. The properties of proportions (see page 245) are used to change proportions into equivalent equations. For example, the product of the extremes equals the product of the means.
4. Similar figures have the same shape. Two polygons are similar if and only if corresponding angles are congruent and corresponding sides are in proportion.
5. Ways to prove two triangles similar:
   AA Similarity Postulate    SAS Similarity Theorem    SSS Similarity Theorem
6. Ways to show that segments are proportional:
   - **a.** Corresponding sides of similar polygons are in proportion.
   - **b.** If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.
   - **c.** If three parallel lines intersect two transversals, they divide the transversals proportionally.
   - **d.** If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.

# Chapter Review

**Write the ratio in simplest form.**

**1.** 15:25    **2.** 6:12:9    **3.** $\frac{16xy}{24x^2}$    7–1

**4.** The measures of the angles of a triangle are in the ratio 4:4:7. Find the three measures.

**Is the equation equivalent to the proportion $\frac{30 - x}{x} = \frac{8}{7}$?**

**5.** $7x = 8(30 - x)$    **6.** $\frac{x}{30 - x} = \frac{7}{8}$    7–2

**7.** $8x = 210 - 7x$    **8.** $\frac{30}{x} = \frac{15}{7}$

**9.** If $\triangle ABC \sim \triangle NJT$, then $\angle B \cong$ _?_. 7–3

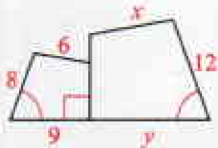
**10.** If quad. $DEFG \sim$ quad. $PQRS$, then $\frac{FG}{RS} = \frac{GD}{?}$.

**11.** $\triangle ABC \sim \triangle JET$, and the scale factor of $\triangle ABC$ to $\triangle JET$ is $\frac{5}{3}$.

**a.** If $BC = 20$, then $ET =$ _?_.

**b.** If the perimeter of $\triangle JET$ is 30, then the perimeter of $\triangle ABC$ is _?_.

**12.** The quadrilaterals are similar. Find the values of $x$ and $y$.




**13. a.** $\triangle RTS \sim$ _?_ 7–4

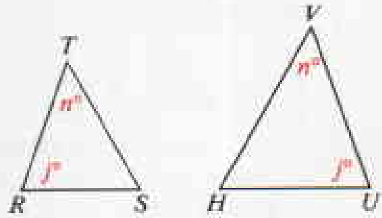
**b.** What postulate or theorem justifies the statement in part (a)?

**14.** $\frac{RT}{?} = \frac{TS}{?} = \frac{RS}{?}$

**15.** Suppose you wanted to prove

$$RS \cdot UV = RT \cdot UH.$$

You would first use similar triangles to show that $\frac{RS}{?} = \frac{?}{?}$.



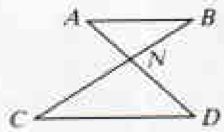

**Can the two triangles be proved similar? If so, state the similarity and the postulate or theorem you would use. If not, write *no*.**

**16.** $\angle A \cong \angle D$ 

**17.** $\angle B \cong \angle D$ 7–5

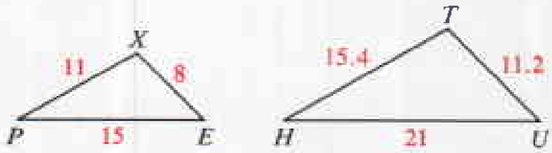
**18.** $CN = 16$, $ND = 14$, $BN = 7$, $AN = 8$

**19.** $AN = 7$, $AB = 13$, $DN = 14$, $DC = 26$




**Exs. 16–19**

**20.**

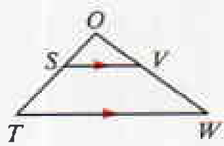



**21.** Which proportion is *incorrect*? 7–6

(1) $\frac{OS}{ST} = \frac{OV}{VW}$ (2) $\frac{SV}{TW} = \frac{OS}{ST}$ (3) $\frac{OT}{OW} = \frac{OS}{OV}$

**22.** If $OS = 8$, $ST = 12$, and $OV = 10$, then $OW =$ _?_.

**23.** If $OS = 8$, $ST = 12$, and $OW = 24$, then $VW =$ _?_.




**24.** In $\triangle ABC$, the bisector of $\angle B$ meets $\overline{AC}$ at $K$. $AB = 18$, $BC = 24$, and $AC = 28$. Find $AK$.

# Chapter Test

**1.** Two sides of a rectangle have the lengths 20 and 32. Find, in simplest form, the ratio of:
**a.** the length of the shorter side to the length of the longer side
**b.** the perimeter to the length of the longer side

**2.** If quad. $ABCD \sim$ quad. $THUS$, then:
**a.** $\angle U \cong$ _?_
**b.** $\frac{BC}{HU} = \frac{AD}{?}$

**3.** If $x:y:z = 4:6:9$ and $z = 45$, then $x =$ _?_ and $y =$ _?_.

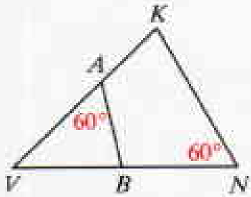
**4.** If $\frac{8}{9} = \frac{x}{15}$, then $x =$ _?_.

**5.** If $\frac{a}{b} = \frac{c}{10}$, then $\frac{a+b}{?} = \frac{?}{10}$.

**6.** What postulate or theorem justifies the statement $\triangle AVB \sim \triangle NVK$?

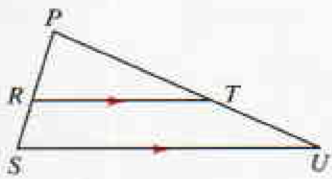
**7.** $\frac{AB}{NK} = \frac{VA}{?}$

**8.** $\angle VBA \cong$ _?_



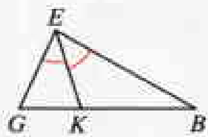

**9.** The scale factor of $\triangle AVB$ to $\triangle NVK$ is $\frac{5}{8}$.
If $VA = 2.5$ and $VB = 1.7$, then $VN =$ _?_.

**10.** If $PR = 10$, $RS = 6$, and $PT = 15$, then $TU =$ _?_.
**11.** If $PT = 32$, $PU = 48$, and $RS = 10$, then $PR =$ _?_.
**12.** If $PR = 14$, $RS = 7$, and $RT = 26$, then $SU =$ _?_.

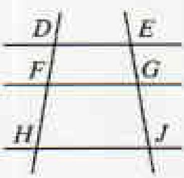



**In $\triangle GEB$, the bisector of $\angle E$ meets $\overline{GB}$ at $K$.**

**13.** If $GK = 5$, $KB = 8$, and $GE = 7$, then $EB =$ _?_.
**14.** If $GE = 14$, $EB = 21$, and $GB = 30$, then $GK =$ _?_.




**15.** Given: $\overleftrightarrow{DE} \parallel \overleftrightarrow{FG} \parallel \overleftrightarrow{HJ}$
Prove: $DF \cdot GJ = FH \cdot EG$




**16.** Given: $BX = 6$; $AX = 8$;
$CX = 9$; $DX = 12$
Prove: $\overline{AB} \parallel \overline{CD}$

# Algebra Review: *Radical Expressions*

The symbol $\sqrt{\phantom{x}}$ always indicates the positive square root of a number. The *radical* $\sqrt{64}$ can be *simplified*.

**Simplify.**

**Example 1** a. $\sqrt{56}$ b. $\sqrt{\frac{16}{3}}$ c. $(3\sqrt{7})^2$

**Solution** a. $\sqrt{56} = \sqrt{4 \cdot 14} = \sqrt{4} \cdot \sqrt{14} = 2\sqrt{14}$

b. $\sqrt{\frac{16}{3}} = \frac{\sqrt{16}}{\sqrt{3}} = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

c. $(3\sqrt{7})^2 = 3\sqrt{7} \cdot 3\sqrt{7} = 3 \cdot 3 \cdot \sqrt{7} \cdot \sqrt{7} = 9 \cdot 7 = 63$

1. $\sqrt{36}$ 2. $\sqrt{81}$ 3. $\sqrt{24}$ 4. $\sqrt{98}$ 5. $\sqrt{300}$

6. $\sqrt{\frac{1}{4}}$ 7. $\frac{\sqrt{5}}{\sqrt{3}}$ 8. $\sqrt{\frac{80}{25}}$ 9. $\frac{2\sqrt{3}}{\sqrt{12}}$ 10. $\sqrt{\frac{250}{48}}$

11. $\sqrt{13^2}$ 12. $(\sqrt{17})^2$ 13. $(2\sqrt{3})^2$ 14. $(3\sqrt{8})^2$ 15. $(9\sqrt{2})^2$

16. $5\sqrt{18}$ 17. $4\sqrt{27}$ 18. $6\sqrt{24}$ 19. $5\sqrt{8}$ 20. $9\sqrt{40}$

**Solve for $x$. Assume $x$ represents a positive number.**

**Example 2** $2^2 + x^2 = 4^2$

**Solution**

$$4 + x^2 = 16$$
$$x^2 = 12$$
$$x = \sqrt{12}$$
$$x = 2\sqrt{3}$$

**Example 3** $x^2 + (3\sqrt{2})^2 = 9^2$

**Solution**

$$x^2 + 18 = 81$$
$$x^2 = 63$$
$$x = \sqrt{63}$$
$$x = 3\sqrt{7}$$

21. $3^2 + 4^2 = x^2$ 22. $x^2 + 4^2 = 5^2$ 23. $5^2 + x^2 = 13^2$

24. $x^2 + 3^2 = 4^2$ 25. $4^2 + 7^2 = x^2$ 26. $x^2 + 5^2 = 10^2$

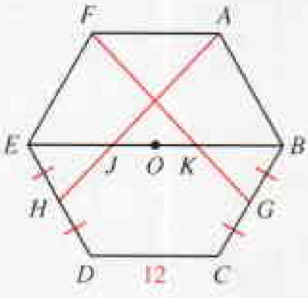
27. $1^2 + x^2 = 3^2$ 28. $x^2 + 5^2 = (5\sqrt{2})^2$ 29. $(x)^2 + (7\sqrt{3})^2 = (2x)^2$

## Challenge

Given regular hexagon $ABCDEF$, with center $O$ and sides of length 12. Let $G$ be the midpoint of $\overline{BC}$. Let $H$ be the midpoint of $\overline{DE}$. $\overline{AH}$ intersects $\overline{EB}$ at $J$ and $\overline{FG}$ intersects $\overline{EB}$ at $K$.

Find $JK$.

(*Hint*: Draw auxiliary lines $\overline{HG}$ and $\overline{DA}$.)



# Cumulative Review: Chapters 1–7

## True-False Exercises

**Write T or F to indicate your answer.**

**A**

1. If $AX = XB$, then $X$ must be the midpoint of $\overline{AB}$.
2. Definitions may be used to justify statements in a proof.
3. If a line and a plane are parallel, then the line is parallel to every line in the plane.
4. When two parallel lines are cut by a transversal, any two angles formed are either congruent or supplementary.
5. If the sides of one triangle are congruent to the corresponding sides of another triangle, then the corresponding angles must also be congruent.
6. Every isosceles trapezoid contains two pairs of congruent angles.

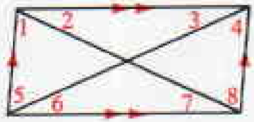
**B**

7. If a quadrilateral has two pairs of supplementary angles, then it must be a parallelogram.
8. If the diagonals of a quadrilateral bisect each other and are congruent, then the quadrilateral must be a square.
9. In $\triangle PQR$, $m\angle P = m\angle R = 50$. If $T$ lies on $\overline{PR}$ and $m\angle PQT = 42$, then $PT < TR$.
10. In quad. $WXYZ$, if $WX = XY = 25$, $YZ = 20$, $ZW = 16$, and $WY = 20$, then $\overline{WY}$ divides the quadrilateral into two similar triangles.
11. Two equiangular hexagons are always similar.

## Multiple-Choice Exercises

**Indicate the best answer by writing the appropriate letter.**

**A**




1. Which pair of angles must be congruent?
   **a.** $\angle 1$ and $\angle 4$ **b.** $\angle 2$ and $\angle 3$
   **c.** $\angle 2$ and $\angle 4$ **d.** $\angle 4$ and $\angle 5$
   **e.** $\angle 2$ and $\angle 8$
2. If $a$, $b$, $c$, and $d$ are coplanar lines such that $a \perp b$, $c \perp d$, and $b \parallel c$, then:
   **a.** $a \perp d$ **b.** $b \parallel d$ **c.** $a \parallel d$ **d.** $a \parallel c$ **e.** none of these
3. If $\triangle ABC \cong \triangle NDH$, then it is also true that:
   **a.** $\angle B \cong \angle H$ **b.** $\angle A \cong \angle H$ **c.** $\overline{AB} \cong \overline{HD}$
   **d.** $\overline{CA} \cong \overline{HN}$ **e.** $\triangle CBA \cong \triangle DHN$

**B**

4. If $PQRS$ is a parallelogram, which of the following *must* be true?
   **a.** $PQ = QR$ **b.** $PQ = RS$ **c.** $PR = QS$ **d.** $\overline{PR} \perp \overline{QS}$ **e.** $\angle Q \cong \angle R$
5. Which of the following can be the lengths of the sides of a triangle?
   **a.** 3, 7, 10 **b.** 3, 7, 11 **c.** 0.5, 7, 7 **d.** $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$ **e.** 1, 3, 5

## Always-Sometimes-Never Exercises

**Write A, S, or N to indicate your choice.**

**A**

1. If a conditional is false, then its converse is _?_ false.
2. Two vertical angles are _?_ adjacent.
3. An angle _?_ has a complement.
4. Two parallel lines are _?_ coplanar.
5. Two perpendicular lines are _?_ both parallel to a third line.
6. A scalene triangle is _?_ equiangular.
7. A regular polygon is _?_ equilateral.
8. A rectangle is _?_ a rhombus.
9. If $\overline{RS} \cong \overline{MN}$, $\overline{ST} \cong \overline{NO}$, and $\angle R \cong \angle M$, then $\triangle RST$ and $\triangle MNO$ are _?_ congruent.
10. The HL method is _?_ appropriate for proving that two acute triangles are congruent.
11. If $AX = BX$, $AY = BY$, and points $A$, $B$, $X$, and $Y$ are coplanar, then $\overline{AB}$ and $\overline{XY}$ are _?_ perpendicular.

**B**

12. The diagonals of a trapezoid are _?_ perpendicular.
13. If a line parallel to one side of a triangle intersects the other two sides, then the triangle formed is _?_ similar to the given triangle.
14. If $\triangle JKL \cong \triangle NET$ and $\overline{NE} \perp \overline{ET}$, then it is _?_ true that $LJ < TE$.
15. If $AB + BC > AC$, then $A$, $B$, and $C$ are _?_ collinear points.
16. A triangle with sides of length $x - 1$, $x$, and $x$ is _?_ an obtuse triangle.

## Completion Exercises

**Complete each statement in the best way.**

**A**

1. If $\overrightarrow{YW}$ bisects $\angle XYZ$ and $m\angle WYX = 60$, then $m\angle XYZ =$ _?_.
2. The acute angles of a right triangle are _?_.
3. A supplement of an acute angle is a(n) _?_ angle.
4. Adjacent angles formed by _?_ lines are congruent.
5. The measure of each interior angle of a regular pentagon is _?_.
6. In $\triangle ABC$ and $\triangle DEF$, $\angle A \cong \angle D$ and $\angle B \cong \angle E$. $\triangle ABC$ and $\triangle DEF$ must be _?_.

**B**

7. When the midpoints of the sides of a rhombus are joined in order, the resulting quadrilateral is best described as a _?_.
8. If $\frac{r}{s} = \frac{t}{u}$, then $\frac{r+s}{t+u} = \frac{?}{?}$.
9. The ratio of the measures of the acute angles of a right triangle is 3:2. The measure of the smaller acute angle is _?_.

## Algebraic Exercises

**In Exercises 1–9 find the value of $x$.**

**A**

**1.** On a number line, $R$ and $S$ have coordinates $-8$ and $x$, and the midpoint of $\overline{RS}$ has coordinate $-1$.

**2.** Two vertical angles have measures $x^2 + 18x$ and $x^2 + 54$.

**3.** The measures of the angles of a quadrilateral are $x$, $x + 4$, $x + 8$, and $x + 12$.

**4.** The lengths of the legs of an isosceles triangle are $7x - 13$ and $2x + 17$.

**5.** Consecutive angles of a parallelogram have measures $6x$ and $2x + 20$.

**6.** A trapezoid has bases of length $x$ and $x + 8$ and a median of length 15.

**7.** $\frac{3x - 1}{4x + 2} = \frac{2}{3}$    **8.** $\frac{5}{8} = \frac{x - 1}{6}$    **9.** $\frac{x}{x + 4} = \frac{x + 3}{x + 9}$

**B**

**10.** The measure of a supplement of an angle is 8 more than three times the measure of a complement. Find the measure of the angle.

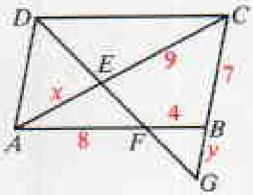
**11.** In a regular polygon, the ratio of the measure of an exterior angle to the measure of an interior angle is 2:13. How many sides does the polygon have?

**12.** The sides of a parallelogram have lengths 12 cm and 15 cm. Find the lengths of the sides of a similar parallelogram with perimeter 90 cm.

**13.** A triangle with perimeter 64 cm has sides with lengths in the ratio 4:5:7. Find the length of each side.

**14.** In $\triangle XYZ$, $XY = YZ$. Find the measure of $\angle Z$ if $m\angle X : m\angle Y = 5:2$.

**15.** In the diagram, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{GC}$. Find the values of $x$ and $y$.

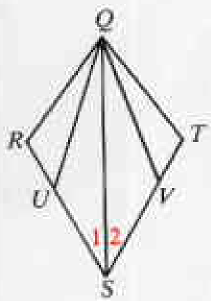



## Proof Exercises

**A**

**1.** Given: $\overline{SU} \cong \overline{SV}$; $\angle 1 \cong \angle 2$
Prove: $\overline{UQ} \cong \overline{VQ}$

**2.** Given: $\overrightarrow{QS}$ bisects $\angle RQT$; $\angle R \cong \angle T$
Prove: $\overrightarrow{SQ}$ bisects $\angle RST$.




**B**

**3.** Given: $\triangle QRU \cong \triangle QTV$; $\overline{US} \cong \overline{VS}$
Prove: $\triangle QRS \cong \triangle QTS$

**4.** Given: $\overline{QS}$ bisects $\angle UQV$ and $\angle USV$; $\angle R \cong \angle T$
Prove: $\overline{RQ} \cong \overline{TQ}$

**5.** Given: $\overline{EF} \parallel \overline{JK}$; $\overline{JK} \parallel \overline{HI}$
Prove: $\triangle EFG \sim \triangle IHG$

**6.** Given: $\frac{JG}{HG} = \frac{KG}{IG}$, $\angle 1 \cong \angle 2$
Prove: $\overline{EF} \parallel \overline{HI}$