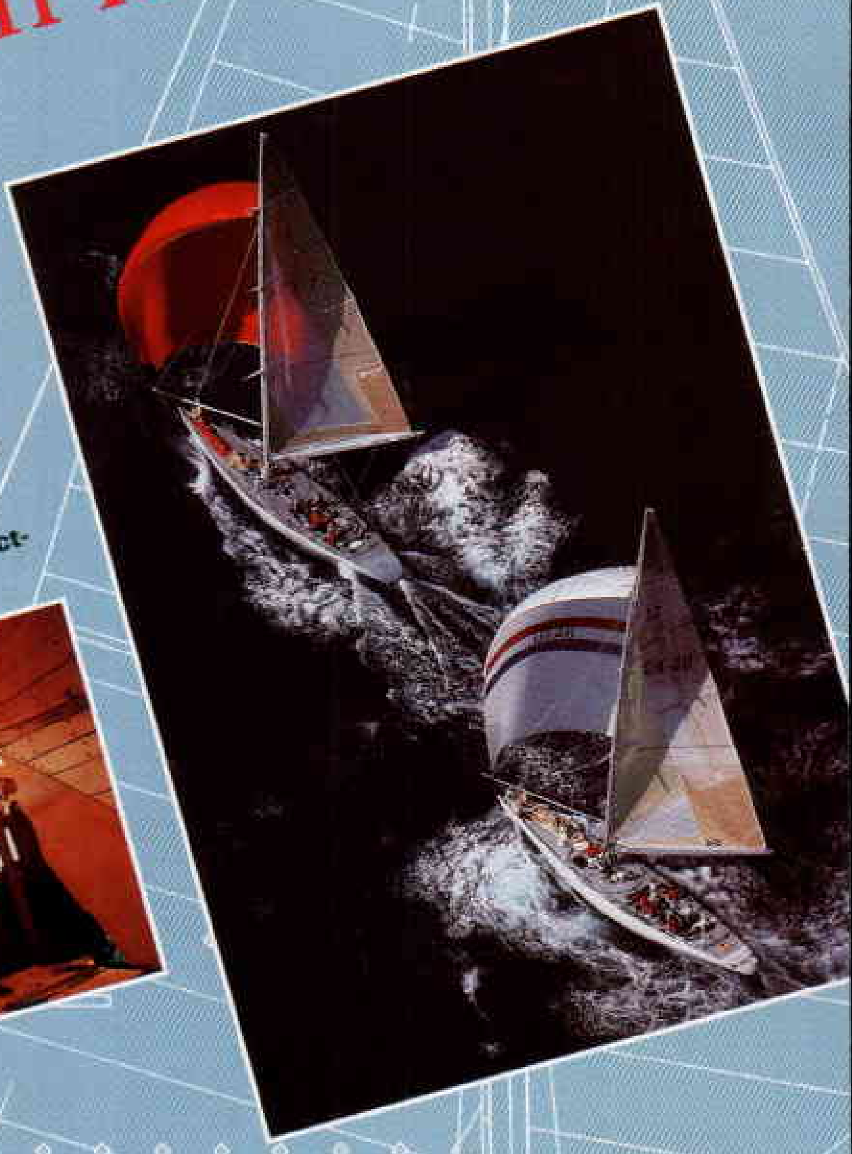
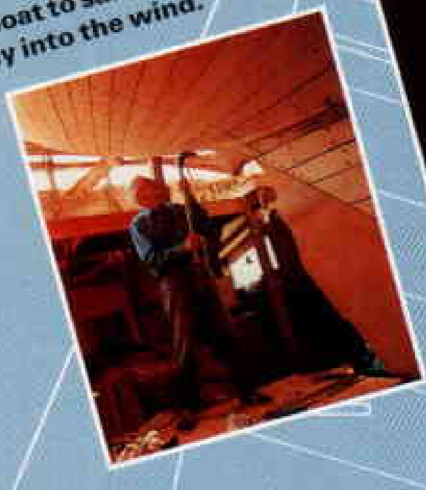
8 RIGHT TRIANGLES

Since the twelfth century sailors have used triangular sails. A triangular sail uses wind power more efficiently than a square sail because it enables a boat to sail more directly into the wind.





Right Triangles

Objectives

1. Determine the geometric mean between two numbers.
2. State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.
3. State and apply the Pythagorean Theorem.
4. State and apply the converse of the Pythagorean Theorem and related theorems about obtuse and acute triangles.
5. Determine the lengths of two sides of a 45°-45°-90° or a 30°-60°-90° triangle when the length of the third side is known.

8-1 *Similarity in Right Triangles*

Recall that in the proportion $\frac{a}{x} = \frac{y}{b}$, the terms shown in red are called the *means*. If a, b, and x are positive numbers and $\frac{a}{x} = \frac{x}{b}$, then x is called the **geometric mean** between a and b. If you solve this proportion for x, you will find that $x = \sqrt{ab}$, a positive number. (The other solution, $x = -\sqrt{ab}$, is discarded because x is defined to be positive.)

Example 1 Find the geometric mean between 5 and 11.

Solution 1 Solve the proportion $\frac{5}{x} = \frac{x}{11}$: $x^2 = 5 \cdot 11$; $x = \sqrt{55}$.

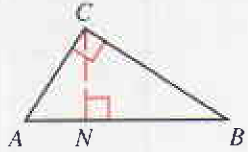
Solution 2 Use the equation $x = \sqrt{ab} = \sqrt{5 \cdot 11} = \sqrt{55}$.

Theorem 8-1

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

Given: $\triangle ABC$ with rt. $\angle ACB$;
altitude $\overline{CN}$

Prove: $\triangle ACB \sim \triangle ANC \sim \triangle CNB$



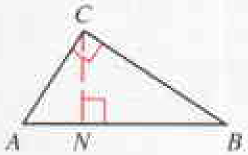
Plan for Proof: Begin by redrawing the three triangles you want to prove similar. Mark off congruent angles and apply the AA Similarity Postulate.

The proof of Theorem 8-1 is left as Exercise 40. The altitude to the hypotenuse divides the hypotenuse into two segments. Corollaries 1 and 2 of Theorem 8-1 deal with geometric means and the lengths of these segments.

For simplicity in stating these corollaries, the words *segment*, *side*, *leg*, and *hypotenuse* are used to refer to the *length* of a segment rather than the segment itself. We will use this convention throughout the book when the context makes this meaning clear.

Corollary 1

When the altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the segments of the hypotenuse.



Given: $\triangle ABC$ with rt. $\angle ACB$; altitude $\overline{CN}$

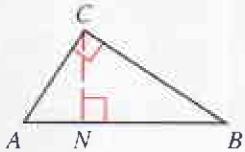
Prove: $\frac{AN}{CN} = \frac{CN}{BN}$

Proof:

By Theorem 8-1, $\triangle ANC \sim \triangle CNB$. Because corresponding sides of similar triangles are in proportion, $\frac{AN}{CN} = \frac{CN}{BN}$.

Corollary 2

When the altitude is drawn to the hypotenuse of a right triangle, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.



Given: $\triangle ABC$ with rt. $\angle ACB$; altitude $\overline{CN}$

Prove: (1) $\frac{AB}{AC} = \frac{AC}{AN}$ and (2) $\frac{AB}{BC} = \frac{BC}{BN}$

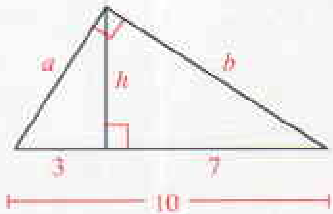
Proof of (1):

By Theorem 8-1, $\triangle ACB \sim \triangle ANC$. Because corresponding sides of similar triangles are in proportion, $\frac{AB}{AC} = \frac{AC}{AN}$. The proof of (2) is very similar.

Example 2 Use the diagram to find the values of h, a, and b.

Solution First determine what parts of the "big" triangle are labeled h, a, and b:

h is the altitude to the hypotenuse,
a is a leg, and b is a leg.



By Corollary 1, $\frac{3}{h} = \frac{h}{7}$ and $h = \sqrt{21}$.

By Corollary 2, $\frac{10}{a} = \frac{a}{3}$ and $a = \sqrt{30}$.

By Corollary 2, $\frac{10}{b} = \frac{b}{7}$ and $b = \sqrt{70}$.

Working with geometric means may involve working with radicals. Radicals should always be written in **simplest form.** This means writing them so that

1. No perfect square factor other than 1 is under the radical sign.
2. No fraction is under the radical sign.
3. No fraction has a radical in its denominator.

Example 3 Simplify: **a.** $5\sqrt{18}$ **b.** $\sqrt{\frac{3}{2}}$ **c.** $\frac{15}{\sqrt{5}}$

Solution

a. Since $18 = 9 \cdot 2$, there is a perfect square factor, 9, under the radical sign.

$$5\sqrt{18} = 5 \cdot \sqrt{9 \cdot 2} = 5 \cdot \sqrt{9} \cdot \sqrt{2} = 5 \cdot 3 \cdot \sqrt{2} = 15\sqrt{2}$$

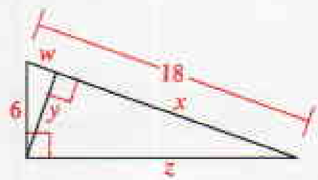
b. There is a fraction, $\frac{3}{2}$, under the radical sign.

$$\sqrt{\frac{3}{2}} = \sqrt{\frac{3}{2} \cdot \frac{2}{2}} = \sqrt{\frac{6}{4}} = \frac{\sqrt{6}}{\sqrt{4}} = \frac{\sqrt{6}}{2}$$

c. There is a radical in the denominator of the fraction.

$$\frac{15}{\sqrt{5}} = \frac{15}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{5} = 3\sqrt{5}$$

Example 4 Find the values of w, x, y, and z.



Solution

$$\frac{18}{6} = \frac{6}{w} \text{ (Cor. 2)}$$
$$18w = 36$$
$$w = 2$$

Then $x = 18 - 2 = 16$.

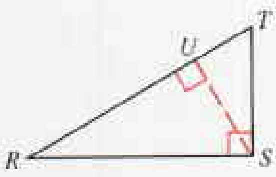
$$\frac{16}{y} = \frac{y}{2} \text{ (Cor. 1)}$$
$$y^2 = 16 \cdot 2$$
$$y = \sqrt{16 \cdot 2}$$
$$y = \sqrt{16} \cdot \sqrt{2}$$
$$y = 4\sqrt{2}$$

$$\frac{18}{z} = \frac{z}{16} \text{ (Cor. 2)}$$
$$z^2 = 16 \cdot 18$$
$$z = \sqrt{16 \cdot 18}$$
$$z = \sqrt{16 \cdot 9 \cdot 2}$$
$$z = 4 \cdot 3 \cdot \sqrt{2} = 12\sqrt{2}$$

Classroom Exercises

Use the diagram to complete each statement.

1. If $m\angle R = 30$, then $m\angle RSU =$ _?_, $m\angle TSU =$ _?_, and $m\angle T =$ _?_.
2. If $m\angle R = k$, then $m\angle RSU =$ _?_, $m\angle TSU =$ _?_, and $m\angle T =$ _?_.
3. $\triangle RST \sim \triangle$ _?_ $\sim \triangle$ _?_
4. $\triangle RSU \sim \triangle$ _?_ $\sim \triangle$ _?_



Exs. 1–4

Simplify.

5. $\sqrt{50}$
6. $3\sqrt{8}$
7. $\sqrt{225}$
8. $7\sqrt{63}$
9. $\sqrt{288}$
10. $\sqrt{\frac{3}{4}}$
11. $\sqrt{\frac{1}{5}}$
12. $\frac{\sqrt{5}}{\sqrt{2}}$
13. $\sqrt{\frac{5}{2}}$
14. $\frac{3}{4}\sqrt{\frac{28}{3}}$
15. Give the geometric mean between:
   a. 2 and 3
   b. 2 and 6
   c. 4 and 25

Study the diagram. Then complete each statement.

16. a. t is the geometric mean between _?_ and _?_.
   b. u is the geometric mean between _?_ and _?_.
   c. v is the geometric mean between _?_ and _?_.

17. a. z is the geometric mean between _?_ and _?_. Thus z = _?_.
   b. x is the geometric mean between _?_ and _?_. Thus x = _?_.
   c. y is the geometric mean between _?_ and _?_. Thus y = _?_.

Written Exercises

Simplify.

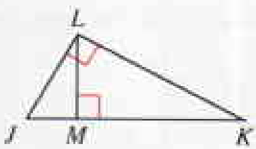
A

1. $\sqrt{12}$
2. $\sqrt{72}$
3. $\sqrt{45}$
4. $\sqrt{75}$
5. $\sqrt{800}$
6. $\sqrt{54}$
7. $9\sqrt{40}$
8. $4\sqrt{28}$
9. $\sqrt{30} \cdot \sqrt{6}$
10. $\sqrt{5} \cdot \sqrt{35}$
11. $\sqrt{\frac{3}{7}}$
12. $\sqrt{\frac{9}{5}}$
13. $\frac{18}{\sqrt{3}}$
14. $\frac{24}{3\sqrt{2}}$
15. $\frac{\sqrt{15}}{3\sqrt{45}}$

Find the geometric mean between the two numbers.

16. 2 and 18
17. 3 and 27
18. 49 and 25
19. 1 and 1000
20. 16 and 24
21. 22 and 55

Exercises 22–30 refer to the figure at the right.



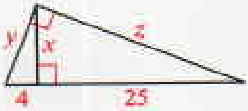
22. If $LM = 4$ and $MK = 8$, find JM.
23. If $LM = 6$ and $JM = 4$, find MK.
24. If $JM = 3$ and $MK = 6$, find LM.
25. If $JM = 4$ and $JK = 9$, find LK.
26. If $JM = 3$ and $MK = 9$, find LJ.

B

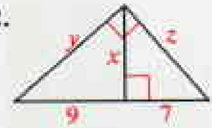
27. If $JM = 3$ and $JL = 6$, find MK.
28. If $JL = 9$ and $JM = 6$, find MK.
29. If $LK = 3\sqrt{6}$ and $MK = 6$, find JM.
30. If $LK = 7$ and $MK = 6$, find JM.

Find the values of *x*, *y*, and *z*.

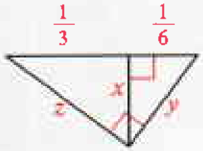
31.



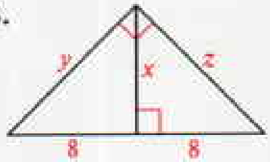
32.



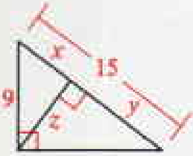
33.



34.

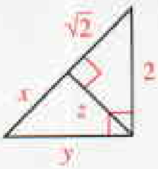


35.

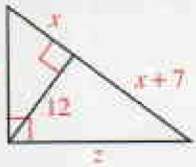


36.

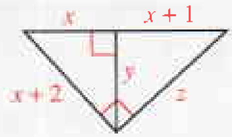
37.



38.



39.

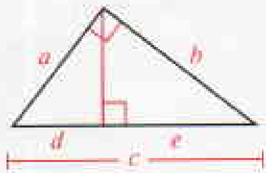


40. Prove Theorem 8-1.

41. **a.** Refer to the figure at the right, and use Corollary 2 to complete:

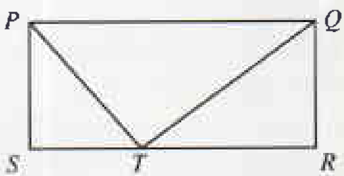
$a^2 = \underline{\ ?\ }$ and $b^2 = \underline{\ ?\ }$

b. Add the equations in part (a), factor the sum on the right, and show that $a^2 + b^2 = c^2$.

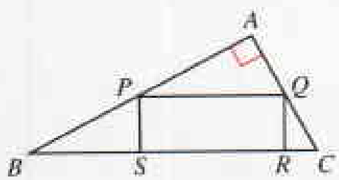


C **42.** Prove: In a right triangle, the product of the hypotenuse and the length of the altitude drawn to the hypotenuse equals the product of the two legs.

43. Given: *PQRS* is a rectangle; *PS* is the geometric mean between *ST* and *TR*.
Prove: $\angle PTQ$ is a right angle.

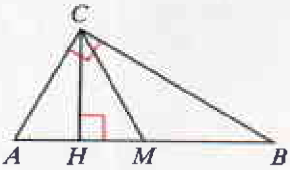


44. Given: *PQRS* is a rectangle; $\angle A$ is a right angle.
Prove: $BS \cdot RC = PS \cdot QR = (PS)^2$



45. The *arithmetic mean* between two numbers r and s is defined to be $\frac{r+s}{2}$.

a. $\overline{CM}$ is the median and $\overline{CH}$ is the altitude to the hypotenuse of right $\triangle ABC$. Show that CM is the arithmetic mean between AH and BH, and that CH is the geometric mean between AH and BH. Then use the diagram to show that the arithmetic mean is greater than the geometric mean.



b. Show algebraically that the arithmetic mean between two different numbers r and s is greater than the geometric mean. (*Hint*: The geometric mean is $\sqrt{rs}$. Work backward from $\frac{r+s}{2} > \sqrt{rs}$ to $(r-s)^2 > 0$ and then reverse the steps.)

8-2 *The Pythagorean Theorem*

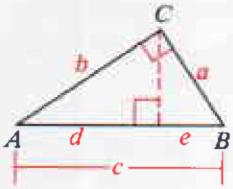
One of the best known and most useful theorems in all of mathematics is the *Pythagorean Theorem*. It is believed that Pythagoras, a Greek mathematician and philosopher, proved this theorem about twenty-five hundred years ago. Many different proofs exist, including one by President Garfield (Exercise 32, page 438) and the proof suggested by the Challenge on page 294.

Theorem 8-2 *Pythagorean Theorem*

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

Given: $\triangle ABC$; $\angle ACB$ is a rt. $\angle$.

Prove: $c^2 = a^2 + b^2$

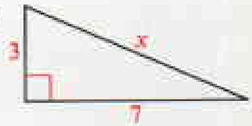


Proof:

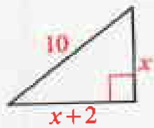
Statements	Reasons
1. Draw a perpendicular from C to $\overline{AB}$.	1. Through a point outside a line, there is exactly one line __?__.
2. $\frac{c}{a} = \frac{a}{e}$; $\frac{c}{b} = \frac{b}{d}$	2. When the altitude is drawn to the hypotenuse of a rt. $\triangle$, each leg is the geometric mean between __?__.
3. $ce = a^2$; $cd = b^2$	3. A property of proportions
4. $ce + cd = a^2 + b^2$	4. Addition Property of =
5. $c(e + d) = a^2 + b^2$	5. Distributive Property
6. $c^2 = a^2 + b^2$	6. Substitution Property

Example Find the value of x. Remember that the length of a segment must be a positive number.

a.



b.



Solution

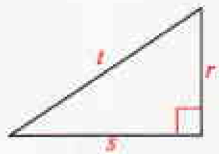
a.
$$x^2 = 7^2 + 3^2$$
$$x^2 = 49 + 9$$
$$x^2 = 58$$
$$x = \sqrt{58}$$

b.
$$x^2 + (x + 2)^2 = 10^2$$
$$x^2 + x^2 + 4x + 4 = 100$$
$$2x^2 + 4x - 96 = 0$$
$$x^2 + 2x - 48 = 0$$
$$(x + 8)(x - 6) = 0$$
~~$x = -8$;~~ $x = 6$

Classroom Exercises

1. The early Greeks thought of the Pythagorean Theorem in this form: *The area of the square on the hypotenuse of a right triangle equals the sum of the areas of the squares on the legs.* Draw a diagram to illustrate that interpretation.

2. Which equations are correct for the right triangle shown?
   a. $r^2 = s^2 + t^2$ **b.** $s^2 = r^2 + t^2$ **c.** $s^2 + r^2 = t^2$
   d. $s^2 = t^2 - r^2$ **e.** $t = r + s$ **f.** $t^2 = (r + s)^2$



Complete each simplification.

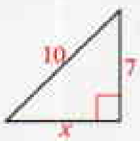
3. $(\sqrt{3})^2 = \sqrt{3} \cdot$ __?__ $=$ __?__

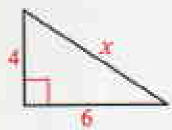
4. $(3\sqrt{11})^2 =$ __?__ $\cdot$ __?__ $= 9 \cdot$ __?__ $=$ __?__

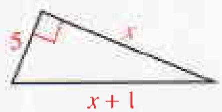
Simplify each expression.

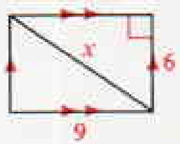
5. $(\sqrt{5})^2$
6. $(2\sqrt{7})^2$
7. $(7\sqrt{2})^2$
8. $(2n)^2$
9. $\left(\frac{3}{\sqrt{5}}\right)^2$
10. $\left(\frac{\sqrt{2}}{2}\right)^2$
11. $\left(\frac{n}{\sqrt{3}}\right)^2$
12. $\left(\frac{2}{3}\sqrt{6}\right)^2$

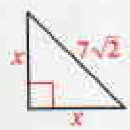
State an equation you could use to find the value of x. Then find the value of x in simplest radical form.

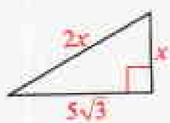
13.


14.


15.


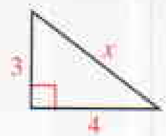
16.


17.


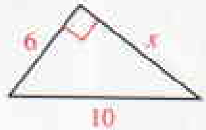
18.


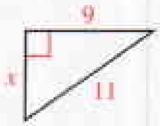
Written Exercises

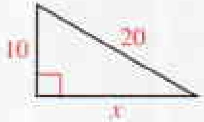
Find the value of *x*.

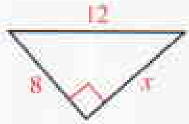
A **1.**


2.

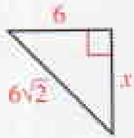

3.


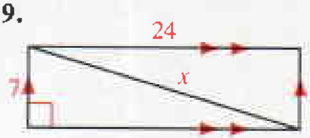
4.


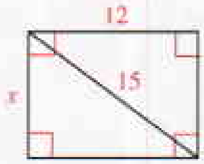
5.


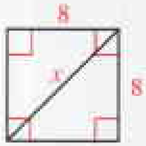
6.


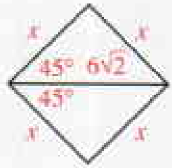
7.


8.


9.


10.


11.


12.


13. A rectangle has length 2.4 and width 1.8. Find the length of a diagonal.

14. A rectangle has a diagonal of 2 and length of $\sqrt{3}$. Find its width.

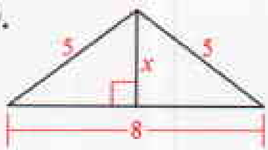
15. Find the length of a diagonal of a square with perimeter 16.

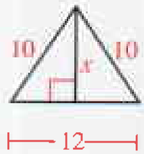
16. Find the length of a side of a square with a diagonal of length 12.

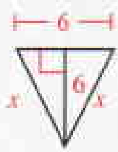
17. The diagonals of a rhombus have lengths 16 and 30. Find the perimeter of the rhombus.

18. The perimeter of a rhombus is 40 cm, and one diagonal is 12 cm long. How long is the other diagonal?

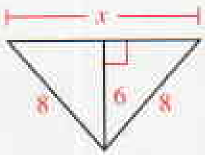
Find the value of *x*.

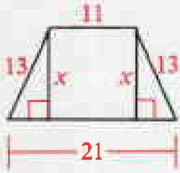
B **19.**


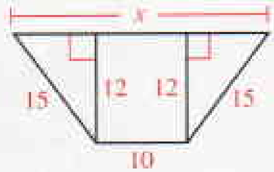
20.


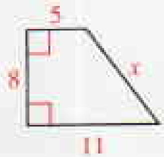
21.


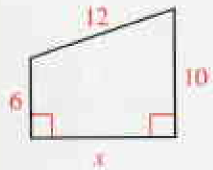
Find the value of x.

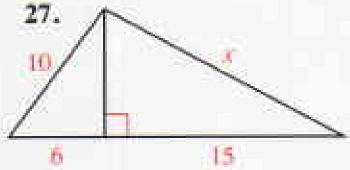
22.


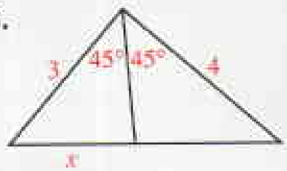
23.


24.


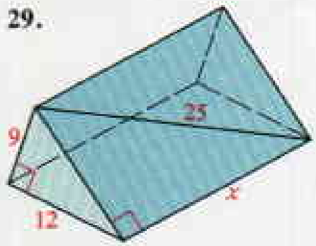
25.


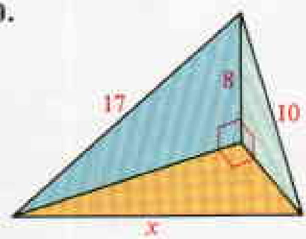
26.


27.


28.


(*Hint*: Use the Angle-Bisector Theorem, p. 270.)

29.


30.


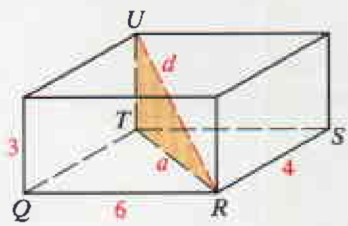
31. A right triangle has legs of 6 and 8. Find the lengths of:
a. the median to the hypotenuse **b.** the altitude to the hypotenuse.

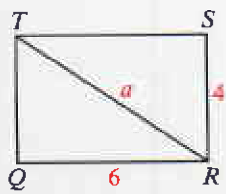
32. A rectangle is 2 cm longer than it is wide. The diagonal of the rectangle is 10 cm long. Find the perimeter of the rectangle.

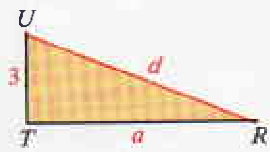
In Exercises 33–36 the dimensions of a rectangular box are given. Sketch the box and find the length of a diagonal of the box.

Example Dimensions 6, 4, 3

Solution







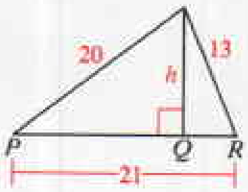
$a^2 = 6^2 + 4^2$
$a^2 = 36 + 16$
$a^2 = 52$

$d^2 = a^2 + 3^2$
$d^2 = 52 + 9$
$d^2 = 61$
$d = \sqrt{61}$

33. 12, 4, 3 **34.** 5, 5, 2 **35.** e, e, e **36.** l, w, h

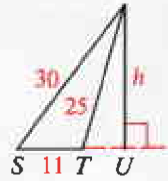
Find the value of h.

C **37.**



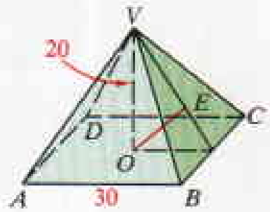
(*Hint*: Let $PQ = x$; $QR = 21 - x$.)

38.



(*Hint*: Let $TU = x$; $SU = x + 11$.)

39. O is the *center* of square $ABCD$ (the point of intersection of the diagonals) and $\overline{VO}$ is perpendicular to the plane of the square. Find OE, the distance from O to the plane of $\triangle VBC$.



Mixed Review Exercises

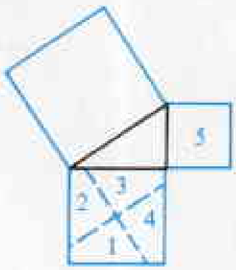
Given: $\triangle ABC$. Complete.

1. If $m\angle A > m\angle B$, then $BC >$ _?_.
2. If $AB > BC$, then $m\angle C$ _?_ $m\angle$ _?_.
3. $AB + BC$ _?_ AC
4. If $\angle C$ is a right angle, then _?_ is the longest side.
5. If $AB = AC$, then $\angle$ _?_ $\cong \angle$ _?_.
6. If $\angle A \cong \angle C$, then $BC =$ _?_.
7. If $\angle C$ is a right angle and X is the midpoint of the hypotenuse, then $AX =$ _?_ $=$ _?_.

Challenge

Start with a right triangle. Build a square on each side. Locate the center of the square drawn on the longer leg. Through the center, draw a parallel to the hypotenuse and a perpendicular to the hypotenuse.

Cut out the pieces numbered 1–5. Can you arrange the five pieces to cover exactly the square built on the hypotenuse? (This suggests another proof of the Pythagorean Theorem.)



8-3 *The Converse of the Pythagorean Theorem*

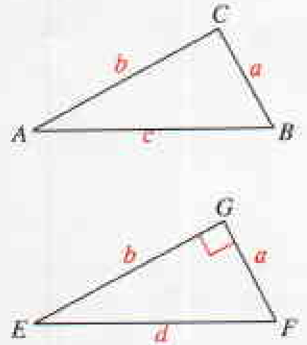
We have seen that the converse of a theorem is not necessarily true. However, the converse of the Pythagorean Theorem *is* true. It is stated below as Theorem 8-3.

Theorem 8-3

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

Given: $\triangle ABC$ with $c^2 = a^2 + b^2$

Prove: $\triangle ABC$ is a right triangle.



Key steps of proof:

1. Draw rt. $\triangle EFG$ with legs a and b.
2. $d^2 = a^2 + b^2$ (Pythagorean Theorem)
3. $c^2 = a^2 + b^2$ (Given)
4. $c = d$ (Substitution)
5. $\triangle ABC \cong \triangle EFG$ (SSS Postulate)
6. $\angle C$ is a rt. $\angle$. (Corr. parts of $\cong$ ⚠ are $\cong$.)
7. $\triangle ABC$ is a rt. $\triangle$. (Def. of a rt. $\triangle$)

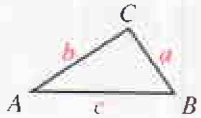
A triangle with sides 3 units, 4 units, and 5 units long is called a 3-4-5 triangle. The numbers 3, 4, and 5 satisfy the equation $a^2 + b^2 = c^2$, so we can apply Theorem 8-3 to conclude that a 3-4-5 triangle is a right triangle. The side lengths shown in the table all satisfy the equation $a^2 + b^2 = c^2$, so the triangles formed are right triangles.

Some Common Right Triangle Lengths

3, 4, 5	5, 12, 13	8, 15, 17	7, 24, 25
6, 8, 10	10, 24, 26		
9, 12, 15			
12, 16, 20			
15, 20, 25			

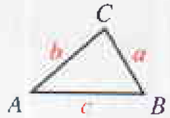
Theorem 8-3 is restated on the next page, along with Theorems 8-4 and 8-5. If you know the lengths of the sides of a triangle, you can use these theorems to determine whether the triangle is right, acute, or obtuse. In each theorem, *c is the length of the longest side* of $\triangle ABC$. Exercises 20 and 19 ask you to state Theorems 8-4 and 8-5 more formally and then prove them.

Theorem 8-3



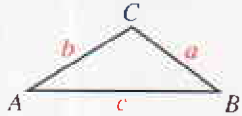
If $c^2 = a^2 + b^2$, then $m\angle C = 90$, and $\triangle ABC$ is right.

Theorem 8-4



If $c^2 < a^2 + b^2$, then $m\angle C < 90$, and $\triangle ABC$ is acute.

Theorem 8-5



If $c^2 > a^2 + b^2$, then $m\angle C > 90$, and $\triangle ABC$ is obtuse.

Example A triangle has sides of the given lengths. Is it acute, right, or obtuse?
a. 9, 40, 41 **b.** 6, 7, 8 **c.** 7, 8, 11

Solution

a. $41^2 \stackrel{?}{=} 9^2 + 40^2$
$1681 \stackrel{?}{=} 81 + 1600$
$1681 = 1681$
The triangle is right.

b. $8^2 \stackrel{?}{=} 6^2 + 7^2$
$64 \stackrel{?}{=} 36 + 49$
$64 < 85$
The triangle is acute.

c. $11^2 \stackrel{?}{=} 7^2 + 8^2$
$121 \stackrel{?}{=} 49 + 64$
$121 > 113$
The triangle is obtuse.

Classroom Exercises

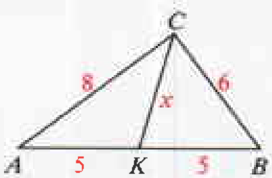
If a triangle is formed with sides having the lengths given, is it acute, right, or obtuse? If a triangle can't be formed, say *not possible*.

1. 6, 8, 10 **2.** 4, 6, 8 **3.** 1, 4, 6
4. 8, 10, 12 **5.** $\sqrt{7}, \sqrt{7}, \sqrt{14}$ **6.** $4, 4\sqrt{3}, 8$

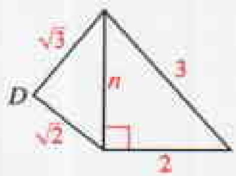
7. Specify all values of x that make the statement true.
a. $\angle 1$ is a right angle. **b.** $\angle 1$ is an acute angle.
c. $\angle 1$ is an obtuse angle. **d.** The triangle is isosceles.
e. No triangle is possible.

Exercises 8–10 refer to the figures below.

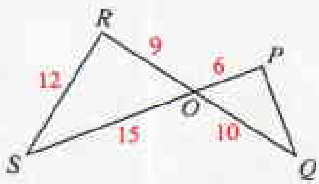
8. Explain why x must equal 5.
9. Explain why $\angle D$ must be a right angle.
10. Explain why $\angle P$ must be a right angle.



Ex. 8



Ex. 9



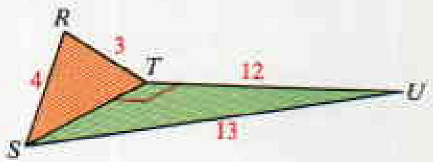
Ex. 10

Written Exercises

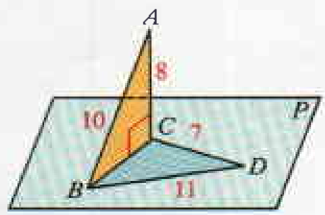
Tell whether a triangle with sides of the given lengths is acute, right, or obtuse.

A

1. 11, 11, 15
2. 9, 9, 13
3. 8, $8\sqrt{3}$, 16
4. 6, 6, $6\sqrt{2}$
5. 8, 14, 17
6. 0.6, 0.8, 1
7. **a.** 0.5, 1.2, 1.3 **b.** $5n$, $12n$, $13n$ where $n > 0$
8. **a.** 33, 44, 55 **b.** $3n$, $4n$, $5n$ where $n > 0$
9. Given: $\angle UTS$ is a rt. $\angle$.
   Show that $\triangle RST$ must be a rt. $\triangle$.



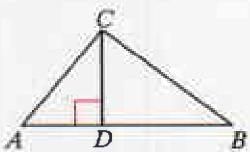
10. Given: $\overline{AC} \perp$ plane P
    Show that $\triangle BCD$ must be obtuse.



Use the information to decide if $\triangle ABC$ is acute, right, or obtuse.

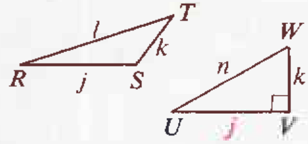
B

11. $AC = 13$, $BC = 15$, $CD = 12$
12. $AC = 10$, $BC = 17$, $CD = 8$
13. $AC = 13$, $BC = \sqrt{34}$, $CD = 3$
14. $AD = 2$, $DB = 8$, $CD = 4$



15. The sides of a triangle have lengths x, $x + 4$, and 20. Specify those values of x for which the triangle is acute with longest side 20.
16. Sketch $\square EFGH$ with $EF = 13$, $EG = 24$, and $FH = 10$. What special kind of parallelogram is $EFGH$? Explain.
17. Sketch $\square RSTU$, with diagonals intersecting at M. $RS = 9$, $ST = 20$, and $RM = 11$. Which segment is longer, $\overline{SM}$ or $\overline{RM}$? Explain.
18. If x and y are positive numbers with $x > y$, show that a triangle with sides of lengths $2xy$, $x^2 - y^2$, and $x^2 + y^2$ is always a right triangle.
19. **a.** Complete this statement of Theorem 8-5:
    If the square of the longest side of a triangle __?__.

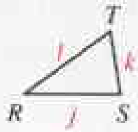
    b. Prove Theorem 8-5.
    Given: $\triangle RST$; $l^2 > j^2 + k^2$
    Prove: $\triangle RST$ is an obtuse triangle.
    (*Hint*: Start by drawing right $\triangle UVW$ with legs j and k. Compare lengths l and n.)

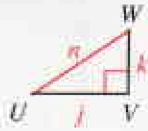


20. a. Complete this statement of Theorem 8-4:
If the square of the longest side of a triangle __?__.

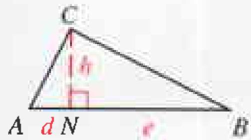
b. Prove Theorem 8-4.

Given: $\triangle RST$; $\overline{RT}$ is the longest side; $l^2 < j^2 + k^2$
Prove: $\triangle RST$ is an acute triangle.
(*Hint*: Start by drawing right $\triangle UVW$ with legs j and k. Compare lengths l and n.)

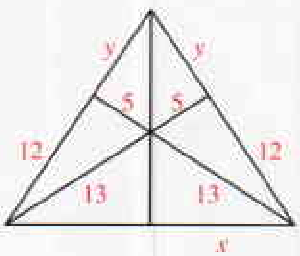


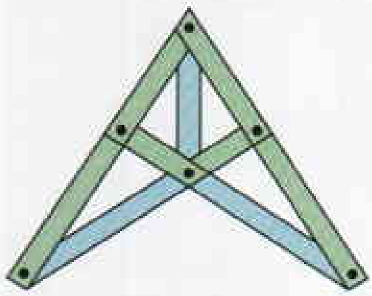


C **21.** Given: $\overline{CN} \perp \overline{AB}$;
h is the geometric mean between d and e.
Prove: $\triangle ABC$ is a right triangle.



22. A frame in the shape of the simple *scissors truss* shown at the right below can be used to support a peaked roof. The weight of the roof compresses some parts of the frame (green), while other parts are in tension (blue). A frame made with s segments joined at j points is stable if $s \geq 2j - 3$. In the truss shown, 9 segments connect 6 points. Verify that the truss is stable. Then find the values of x and y.





Explorations

These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.

The sides of a quadrilateral have lengths a, b, c, and d. The diagonals have lengths e and f. For what kinds of quadrilaterals does

$$a^2 + b^2 + c^2 + d^2 = e^2 + f^2?$$

Draw various quadrilaterals including a parallelogram, rectangle, rhombus, trapezoid, and a random quadrilateral.

♦ Computer Key-In

Suppose a, b, and c are positive integers such that $a^2 + b^2 = c^2$. Then the converse of the Pythagorean Theorem guarantees that a, b, and c are the lengths of the sides of a right triangle. Because of this, any such triple of integers is called a **Pythagorean triple.**

For example, 3, 4, 5 is a Pythagorean triple since $3^2 + 4^2 = 5^2$. Another triple is 6, 8, 10, since $6^2 + 8^2 = 10^2$. The triple 3, 4, 5 is called a *primitive* Pythagorean triple because no factor (other than 1) is common to all three integers. The triple 6, 8, 10 is *not* a primitive triple.

The following program in BASIC lists some Pythagorean triples.

```
10  FOR X = 2 TO 7
20    FOR Y = 1 TO X - 1
30      LET A = 2 * X * Y
40      LET B = X * X - Y * Y
50      LET C = X * X + Y * Y
60      PRINT A;",";B;",";C
70    NEXT Y
80  NEXT X
90  END
```

Exercises

1. Type and RUN the program. (If your computer uses a language other than BASIC, write and RUN a similar program.) What Pythagorean triples did it list? Which of these are primitive Pythagorean triples?

2. The program above uses a method for finding Pythagorean triples that was developed by Euclid around 320 B.C. His method can be stated as follows:

> If x and y are positive integers with $y < x$, then $a = 2xy$, $b = x^2 - y^2$, and $c = x^2 + y^2$ is a Pythagorean triple.

To verify that Euclid's method is correct, show that the equation below is true.

$$(2xy)^2 + (x^2 - y^2)^2 = (x^2 + y^2)^2$$

3. Look at the primitive Pythagorean triples found in Exercise 1. List those triples that have an odd number as their lowest value. Do you notice a pattern in some of these triples?

Another method for finding Pythagorean triples begins with an odd number. If n is any positive integer, $2n + 1$ is an odd number. A triple is given by: $a = 2n + 1$, $b = 2n^2 + 2n$, $c = (2n^2 + 2n) + 1$.
For example, when $n = 3$, the triple is

$$2(3) + 1 = 7, \quad 2(3^2) + 2(3) = 24, \quad 24 + 1 = 25.$$

a. Use the formula to find another primitive triple with 33 as its lowest value. (*Hint*: $n = 16$)

b. Use the Pythagorean Theorem to verify the method described.

8-4 *Special Right Triangles*

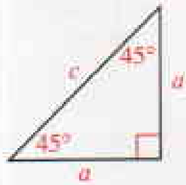
An isosceles right triangle is also called a 45°-45°-90° triangle, because the measures of the angles are 45, 45, and 90.

Theorem 8-6 *45°-45°-90° Theorem*

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.

Given: A 45°-45°-90° triangle

Prove: hypotenuse = $\sqrt{2} \cdot$ leg



Plan for Proof: Let the sides of the given triangle be a, a, and c. Apply the Pythagorean Theorem and solve for c in terms of a.

Example 1 Find the value of x.

a.

b.

Solution

a. hyp = $\sqrt{2} \cdot$ leg

$x = \sqrt{2} \cdot 12$

$x = 12\sqrt{2}$

b. hyp = $\sqrt{2} \cdot$ leg

$8 = \sqrt{2} \cdot x$

$$x = \frac{8}{\sqrt{2}} = \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2}$$

$x = 4\sqrt{2}$

Another special right triangle has acute angles measuring 30 and 60.

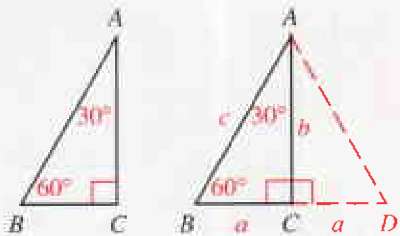
Theorem 8-7 *30°-60°-90° Theorem*

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

Given: $\triangle ABC$, a 30°-60°-90° triangle

Prove: hypotenuse = $2 \cdot$ shorter leg

longer leg = $\sqrt{3} \cdot$ shorter leg

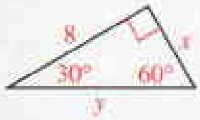


Plan for Proof: Build onto $\triangle ABC$ as shown. $\triangle ADC \cong \triangle ABC$, so $\triangle ABD$ is equiangular and equilateral with $c = 2a$. Since $\triangle ABC$ is a right triangle, $a^2 + b^2 = c^2$. By substitution, $a^2 + b^2 = 4a^2$, so $b^2 = 3a^2$ and $b = a\sqrt{3}$.

Example 2 Find the values of x and y.

a.

b.



Solution

a. hyp. = 2 · shorter leg

$$x = 2 \cdot 6$$
$$x = 12$$

longer leg = $\sqrt{3}$ · shorter leg

$$y = 6\sqrt{3}$$

b. longer leg = $\sqrt{3}$ · shorter leg

$$8 = \sqrt{3} \cdot x$$
$$x = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$$

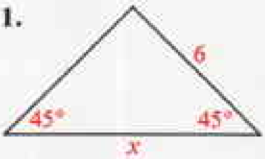
hyp. = 2 · shorter leg

$$y = 2 \cdot \frac{8\sqrt{3}}{3} = \frac{16\sqrt{3}}{3}$$

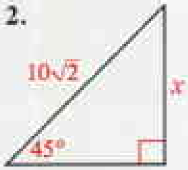
Classroom Exercises

Find the value of x.

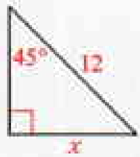
1.



2.



3.

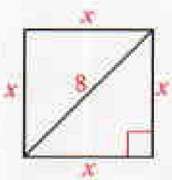


4.

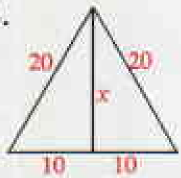
5.

6.

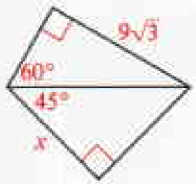
7.



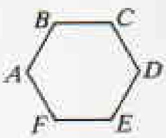
8.



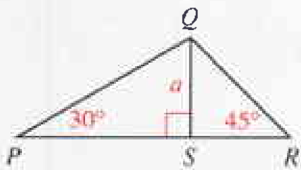
9.



10. In regular hexagon $ABCDEF$, $AB = 8$. Find AD and AC.



11. Express PQ, PS, and QR in terms of a.



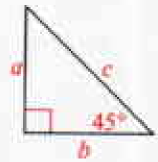
12. If the measures of the angles of a triangle are in the ratio 1:2:3, are the lengths of the sides in the same ratio? Explain.

Written Exercises

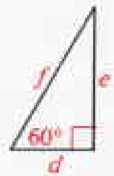
Copy and complete the tables.

A

	1.	2.	3.	4.	5.	6.	7.	8.
a	4	?	$\sqrt{5}$	?	?	?	?	?
b	?	$\frac{2}{3}$	?	?	?	?	$4\sqrt{2}$	?
c	?	?	?	$3\sqrt{2}$	6	$\sqrt{14}$	?	5



	9.	10.	11.	12.	13.	14.	15.	16.
d	7	$\frac{1}{4}$	?	?	?	?	?	?
e	?	?	$5\sqrt{3}$	6	?	?	3	?
f	?	?	?	?	10	13	?	$6\sqrt{3}$



17. Find the length of a diagonal of a square whose perimeter is 48.

18. A diagonal of a square has length 8. What is the perimeter of the square?

19. An altitude of an equilateral triangle has length $6\sqrt{3}$. What is the perimeter of the triangle?

20. Find the altitude of an equilateral triangle if each side is 10 units long.

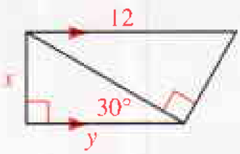
Find the values of x and y in each diagram.

B **21.**

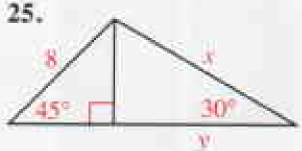
22.

23.

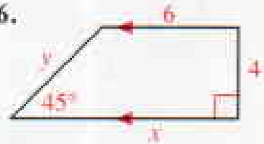
24.



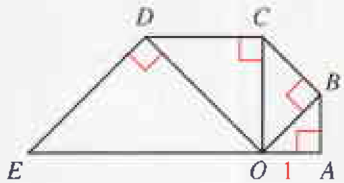
25.



26.



27. The diagram shows four 45°-45°-90° triangles. If $OA = 1$, find OB, OC, OD, and OE.



28. The diagonals of a rectangle are 8 units long and intersect at a 60° angle. Find the dimensions of the rectangle.

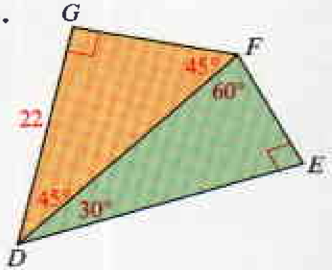
29. The perimeter of a rhombus is 64 and one of its angles has measure 120. Find the lengths of the diagonals.

30. Prove Theorem 8-6.

31. Explain why any triangle having sides in the ratio $1:\sqrt{3}:2$ must be a 30°-60°-90° triangle.

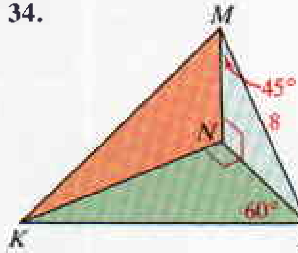
Find the lengths of as many segments as possible.

32.



33.

34.



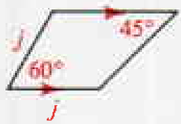
C **35.** In quadrilateral $QRST$, $m\angle R = 60$, $m\angle T = 90$, $QR = RS$, $ST = 8$, and $TQ = 8$.

a. How long is the longer diagonal of the quadrilateral?

b. Find the ratio of RT to QS.

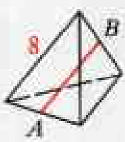
36. Find the perimeter of the triangle.

37. Find the length of the median of the trapezoid in terms of j.



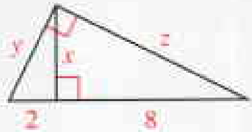
38. If the wrench just fits the hexagonal nut, what is the value of x?

★ **39.** The six edges of the solid shown are 8 units long. A and B are midpoints of two edges as shown. Find AB.



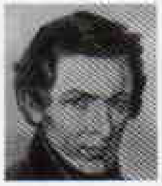
Self-Test 1

1. Find the geometric mean between 3 and 15.
2. The diagram shows the altitude drawn to the hypotenuse of a right triangle.
   a. x = ?
   b. y = ?
   c. z = ?



3. The sides of a triangle are given. Is the triangle acute, right, or obtuse?
   a. 11, 60, 61 b. 7, 9, 11 c. 0.2, 0.3, 0.4
4. A rectangle has length 8 and width 4. Find the lengths of the diagonals.
5. Find the perimeter of a square that has diagonals 10 cm long.
6. The sides of an equilateral triangle are 12 cm long. Find the length of an altitude of the triangle.
7. How long is the altitude to the base of an isosceles triangle if the sides of the triangle are 13, 13, and 10?

Biographical Note — *Nikolai Lobachevsky*



Lobachevsky (1793–1856) was a Russian mathematician who brought a new insight to the study of geometry. He realized that Euclidean geometry is only one geometry, and that other geometric systems are possible.

A modern restatement of Euclid's fifth postulate, often called the Parallel Postulate, is "Through a point outside a line, there is exactly one line parallel to the given line." It is this postulate that defines *Euclidean* geometry. For 2000 years, mathematicians tried to prove this fifth postulate from the other four.

Lobachevsky tried a different approach. He created a geometric system where Euclid's first four postulates were the same but the fifth was changed to allow *more than one* parallel through a given point. The antique model at the left shows such a system. Other geometric systems based on a different fifth postulate followed. (See Extra: Non-Euclidean Geometries, page 233.)

Although Lobachevsky thought our universe was Euclidean, some physicists have decided the universe may be better described by Lobachevsky's system. Even so, over small regions Euclidean geometry is accurate. Similarly, although the surface of the Earth is a sphere, we treat small areas of it as flat.

Trigonometry

Objectives

1. Define the tangent, sine, and cosine ratios for an acute angle.
2. Solve right triangle problems by correct selection and use of the tangent, sine, and cosine ratios.

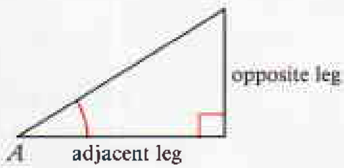
8-5 *The Tangent Ratio*

The word trigonometry comes from Greek words that mean "triangle measurement." In this book our study will be limited to the trigonometry of right triangles. In the right triangle shown, one acute angle is marked. The leg opposite this angle and the leg adjacent to this angle are labeled.

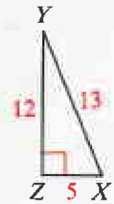
The following ratio of the lengths of the legs is called the *tangent ratio*.

tangent of $\angle A = \dfrac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$

In abbreviated form: $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$



Example 1 Find tan X and tan Y.



Solution

$$\tan X = \frac{\text{leg opposite } \angle X}{\text{leg adjacent to } \angle X} = \frac{12}{5}$$

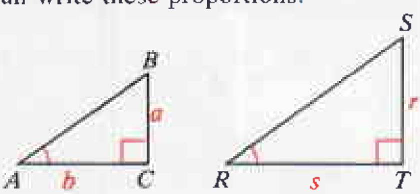
$$\tan Y = \frac{\text{leg opposite } \angle Y}{\text{leg adjacent to } \angle Y} = \frac{5}{12}$$

In the right triangles shown below, $m\angle A = m\angle R$. Then by the AA Similarity Postulate, the triangles are similar. We can write these proportions:

$\dfrac{a}{r} = \dfrac{b}{s}$ (Why?)

$\dfrac{a}{b} = \dfrac{r}{s}$ (A property of proportions)

$\tan A = \tan R$ (Def. of tangent ratio)



We have shown that if $m\angle A = m\angle R$, then $\tan A = \tan R$. Thus, we have shown that the value of the tangent of an angle depends only on the size of the angle, not on the size of the right triangle. It is also true that if $\tan A = \tan R$ for acute angles A and R, then $m\angle A = m\angle R$.

Since the tangent of an angle depends only on the measure of the angle, we can write tan 10°, for example, to stand for the tangent of any angle with a degree measure of 10. The table on page 311 lists the values of the tangents of some angles with measures between 0 and 90. Most of the values are approximations, rounded to four decimal places. Suppose you want the approximate value of tan 33°. Locate 33° in the angle column. Go across to the tangent column. Read .6494. You write tan 33° ≈ 0.6494, where the symbol ≈ means "is approximately equal to." You can also use a scientific calculator to find tan 33° ≈ 0.649407593. Your calculator may give more or fewer decimal places than the nine that are shown.

Example 2 Find the value of y to the nearest tenth.

Solution

$$\tan 56° = \frac{y}{32}$$
$$y = 32(\tan 56°)$$
$$y \approx 32(1.4826)$$
$$y \approx 47.4432, \text{ or } 47.4$$

You can find the approximate degree measure of an angle with a given tangent by reading the table from the tangent column across to the angle column, or by using the inverse tangent key(s) of a calculator.

Example 3 The grade of a road is the ratio of its rise to its run and is usually given as a decimal or percent. Find the angle that the road makes with the horizontal if its grade is 4% ($\frac{4}{100}$ or 0.04).

Solution

$$\tan x° = 0.0400$$
$$x° \approx 2°$$

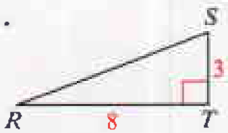
$$\text{grade} = \frac{\text{rise}}{\text{run}}$$

If you use the table on page 311, notice that 0.0400 falls between two values in the tangent column: tan 2° ≈ 0.0349 and tan 3° ≈ 0.0524. Since 0.0349 is closer to 0.0400, we use 2° as an approximate value for $x°$.

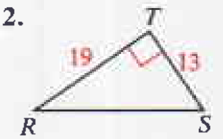
Classroom Exercises

In Exercises 1–3 express tan *R* as a ratio.

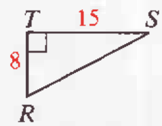
1.



2.



3.



4–6. Express tan S as a ratio for each triangle above.

7. Use the table on page 311 to complete the statements.

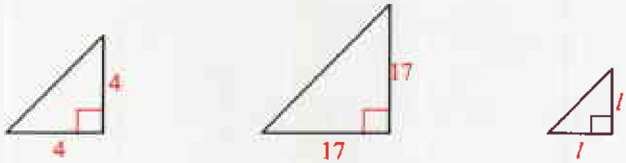
a. tan 24° ≈ ___?___ **b.** tan 41° ≈ ___?___ **c.** tan 88° ≈ ___?___

d. tan ___?___ ≈ 2.4751 **e.** tan ___?___ ≈ 0.3057 **f.** tan ___?___ ≈ 0.8098

8. Three 45°-45°-90° triangles are shown below.

a. In each triangle, express tan 45° in simplified form.

b. See the entry for tan 45° on page 311. Is the entry exact?

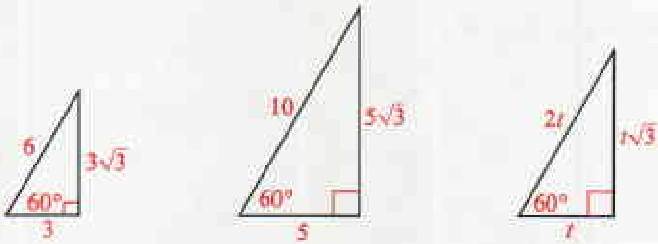


9. Three 30°-60°-90° triangles are shown below.

a. In each triangle, express tan 60° in simplified radical form.

b. Use $\sqrt{3} \approx 1.732051$ to find an approximate value for tan 60°.

c. Is the entry for tan 60° on page 311 exact? Is it correct to four decimal places?



10. Notice that the tangent values increase rapidly toward the end of the table on page 311. Explain how you know that there is some angle with a tangent value equal to 1,000,000. Is there any upper limit to tangent values?

11. Two ways to find the value of x are started below.

Using tan 40°:

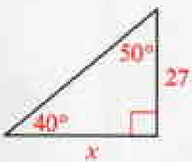
$$\tan 40° = \frac{27}{x}$$

$$0.8391 \approx \frac{27}{x}$$

Using tan 50°:

$$\tan 50° = \frac{x}{27}$$

$$1.1918 \approx \frac{x}{27}$$



Which of the following statements are correct?

a. $x \approx 27 \cdot 0.8391$

b. $x \approx 27 \cdot 1.1918$

c. $x \approx \dfrac{27}{0.8391}$

d. $x \approx \dfrac{27}{1.1918}$

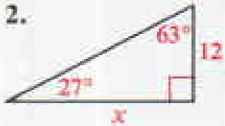
Which correct statement is easier to use for computing if you are *not* using a calculator for the arithmetic?

Written Exercises

Find the value of x to the nearest tenth. Use a calculator or the table on page 311.

A **1.**

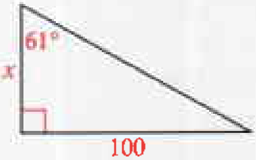
2.



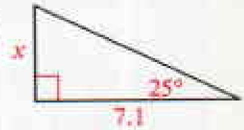
3.

4.

5.

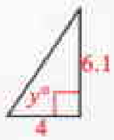


6.



Find $y°$ correct to the nearest degree.

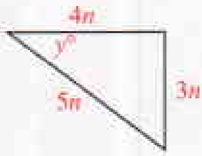
7.



8.

9.

10.

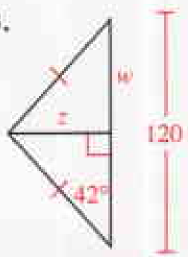


11.

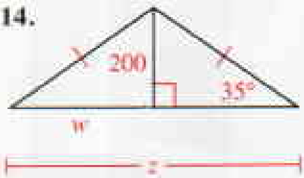
12.

Find w, then z, correct to the nearest integer.

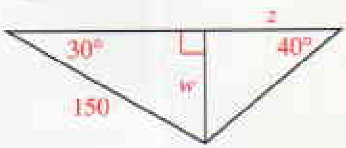
B **13.**



14.

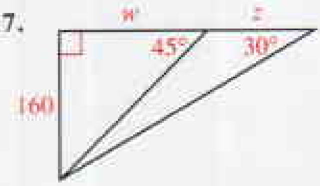


15.

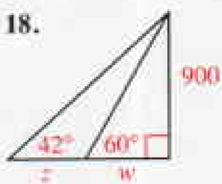


16.

17.



18.



19. The grade of a road is 7%. What angle does the road make with the horizontal?

20. A road climbs at an 8° angle with the horizontal. What is the grade of the road?

21. The base of an isosceles triangle is 70 cm long. The altitude to the base is 75 cm long. Find, to the nearest degree, the base angles of the triangle.

22. A rhombus has diagonals of length 4 and 10. Find the angles of the rhombus to the nearest degree.

23. The shorter diagonal of a rhombus with a 70° angle is 122 cm long. How long, to the nearest centimeter, is the longer diagonal?

24. A rectangle is 80 cm long and 20 cm wide. Find, to the nearest degree, the acute angle formed at the intersection of the diagonals.

25. A natural question to consider is the following:

$$\text{Does } \tan A + \tan B = \tan (A + B)?$$

Try substituting 35° for A and 25° for B.

a. $\tan 35° + \tan 25° \approx$ _?_ + _?_ = _?_

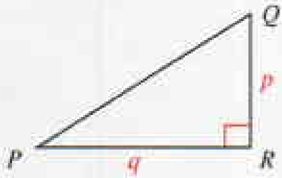
b. $\tan (35° + 25°) = \tan$ _?_ ° $\approx$ _?_

c. What is your answer to the general question raised in this exercise, *yes* or *no*?

d. Do you think $\tan A - \tan B = \tan (A - B)$? Explain.

26. a. Given: $\triangle PQR$; $\angle R$ is a right angle.
Prove: $\tan P \cdot \tan Q = 1$

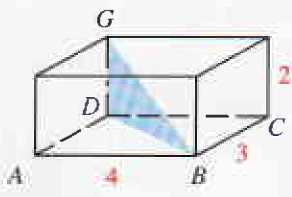
b. If $\tan 32° \approx \frac{5}{8}$, find $\tan 58°$ without using a table or a calculator.



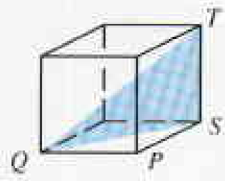
27. A rectangular box has length 4, width 3, and height 2.

a. Find BD.

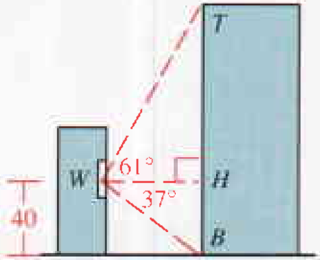
b. Find $\angle GBD$ to the nearest degree.



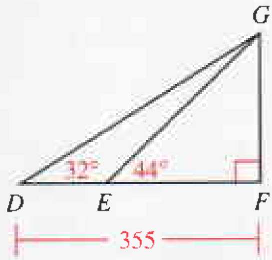
C **28.** If the figure is a cube, find $\angle TQS$ to the nearest degree.



29. A person at window W, 40 ft above street level, sights points on a building directly across the street. H is chosen so that $\overline{WH}$ is horizontal. T is directly above H, and B is directly below. By measurement, $m\angle TWH = 61$ and $m\angle BWH = 37$. How far above street level is T?



Ex. 29



Ex. 30

30. Use the figure to find EF to the nearest integer.

Explorations

These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.

As you will learn in the next section, two other trigonometric ratios are the *sine* and *cosine*. If $\triangle ABC$ has a right angle at B, then:

$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$

Using ASA, draw nine right triangles using nine values for $m\angle A$: 10, 20, 30, 40, 45, 50, 60, 70, and 80. Keep $m\angle B = 90$.

Compute and record sin A, cos A, and tan A for each measure of $\angle A$. What do you notice?

If you change the length of $\overline{AB}$ but keep the measures of $\angle A$ and $\angle B$ the same, do the sine, cosine, and tangent of $\angle A$ change?

Complete.

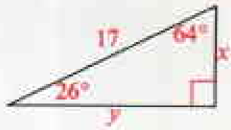
1. $\cos x° = \sin x°$ when $x =$ _?_
2. $\cos (90 - x)° = \sin$ _?_
3. $\sin (90 - x)° = \cos$ _?_
4. $\tan x° \cdot \tan (90 - x)° =$ _?_
5. For acute angles, what trigonometric ratios have values between 0 and 1?
6. What trigonometric ratio can have values greater than 1?

Table of Trigonometric Ratios

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175	46°	.7193	.6947	1.0355
2°	.0349	.9994	.0349	47°	.7314	.6820	1.0724
3°	.0523	.9986	.0524	48°	.7431	.6691	1.1106
4°	.0698	.9976	.0699	49°	.7547	.6561	1.1504
5°	.0872	.9962	.0875	50°	.7660	.6428	1.1918
6°	.1045	.9945	.1051	51°	.7771	.6293	1.2349
7°	.1219	.9925	.1228	52°	.7880	.6157	1.2799
8°	.1392	.9903	.1405	53°	.7986	.6018	1.3270
9°	.1564	.9877	.1584	54°	.8090	.5878	1.3764
10°	.1736	.9848	.1763	55°	.8192	.5736	1.4281
11°	.1908	.9816	.1944	56°	.8290	.5592	1.4826
12°	.2079	.9781	.2126	57°	.8387	.5446	1.5399
13°	.2250	.9744	.2309	58°	.8480	.5299	1.6003
14°	.2419	.9703	.2493	59°	.8572	.5150	1.6643
15°	.2588	.9659	.2679	60°	.8660	.5000	1.7321
16°	.2756	.9613	.2867	61°	.8746	.4848	1.8040
17°	.2924	.9563	.3057	62°	.8829	.4695	1.8807
18°	.3090	.9511	.3249	63°	.8910	.4540	1.9626
19°	.3256	.9455	.3443	64°	.8988	.4384	2.0503
20°	.3420	.9397	.3640	65°	.9063	.4226	2.1445
21°	.3584	.9336	.3839	66°	.9135	.4067	2.2460
22°	.3746	.9272	.4040	67°	.9205	.3907	2.3559
23°	.3907	.9205	.4245	68°	.9272	.3746	2.4751
24°	.4067	.9135	.4452	69°	.9336	.3584	2.6051
25°	.4226	.9063	.4663	70°	.9397	.3420	2.7475
26°	.4384	.8988	.4877	71°	.9455	.3256	2.9042
27°	.4540	.8910	.5095	72°	.9511	.3090	3.0777
28°	.4695	.8829	.5317	73°	.9563	.2924	3.2709
29°	.4848	.8746	.5543	74°	.9613	.2756	3.4874
30°	.5000	.8660	.5774	75°	.9659	.2588	3.7321
31°	.5150	.8572	.6009	76°	.9703	.2419	4.0108
32°	.5299	.8480	.6249	77°	.9744	.2250	4.3315
33°	.5446	.8387	.6494	78°	.9781	.2079	4.7046
34°	.5592	.8290	.6745	79°	.9816	.1908	5.1446
35°	.5736	.8192	.7002	80°	.9848	.1736	5.6713
36°	.5878	.8090	.7265	81°	.9877	.1564	6.3138
37°	.6018	.7986	.7536	82°	.9903	.1392	7.1154
38°	.6157	.7880	.7813	83°	.9925	.1219	8.1443
39°	.6293	.7771	.8098	84°	.9945	.1045	9.5144
40°	.6428	.7660	.8391	85°	.9962	.0872	11.4301
41°	.6561	.7547	.8693	86°	.9976	.0698	14.3007
42°	.6691	.7431	.9004	87°	.9986	.0523	19.0811
43°	.6820	.7314	.9325	88°	.9994	.0349	28.6363
44°	.6947	.7193	.9657	89°	.9998	.0175	57.2900
45°	.7071	.7071	1.0000				

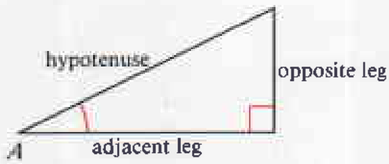
8-6 *The Sine and Cosine Ratios*

Suppose you want to find the legs, x and y, in the triangle at the right. You can't easily find these values using the tangent ratio because the only side you know is the hypotenuse. The ratios that relate the legs to the hypotenuse are the *sine* and *cosine*.



sine of $\angle A = \dfrac{\text{leg opposite } \angle A}{\text{hypotenuse}}$

cosine of $\angle A = \dfrac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$



We now have three useful trigonometric ratios, given below in abbreviated form:

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Example 1 Find the values of x and y to the nearest integer.

Solution

$\sin 67° = \dfrac{x}{120}$

$x = 120 \cdot \sin 67°$

$x \approx 120(0.9205)$

$x \approx 110.46$, or 110

$\cos 67° = \dfrac{y}{120}$

$y = 120 \cdot \cos 67°$

$y \approx 120(0.3907)$

$y \approx 46.884$, or 47

Example 2 Find the value of n to the nearest integer.

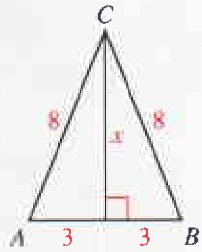
Solution

$\sin n° = \dfrac{22}{40}$

$\sin n° = 0.5500$

$n \approx 33$

Example 3 An isosceles triangle has sides 8, 8, and 6. Find the lengths of its three altitudes.

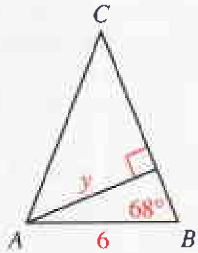


Solution The altitude to the base can be found using the Pythagorean Theorem.

$x^2 = 8^2 - 3^2 = 55$

$x = \sqrt{55} \approx 7.4$

Notice that $\cos B = \frac{3}{8}$ (so $m\angle B \approx 68$), and that the altitudes from A and B are congruent. (Why?)
To find the length of the altitudes from A and B, use



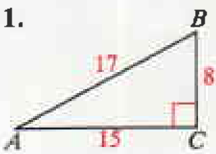
$$\sin B \approx \sin 68° \approx \frac{y}{6}.$$

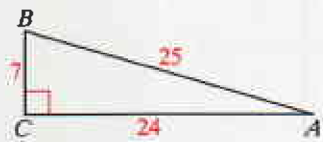
$$y \approx 6 \cdot \sin 68°$$

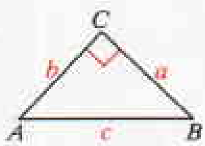
$$y \approx 5.6$$

Classroom Exercises

In Exercises 1–3 express sin *A*, cos *A*, and tan *A* as fractions.

1. 

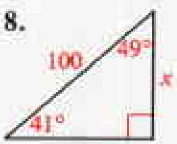
2. 

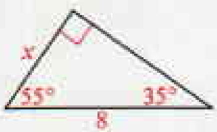
3. 

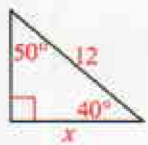
4–6. Using the triangles in Exercises 1–3, express sin B, cos B, and tan B as fractions.

7. Use the table on page 311 or a scientific calculator to complete the statements.
   a. $\sin 24° \approx$ __?__ **b.** $\cos 57° \approx$ __?__ **c.** $\sin 87° \approx$ __?__
   d. cos __?__ ≈ 0.9659 **e.** sin __?__ ≈ 0.1045 **f.** cos __?__ ≈ 0.1500

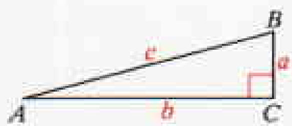
State two different equations you could use to find the value of *x*.

8. 

9. 

10. 

11. The word *cosine* is related to the phrase "complement's sine." Explain the relationship by using the diagram to express the cosine of $\angle A$ and the sine of its complement, $\angle B$.



12. The table on page 311 lists 0.5000 as the value of sin 30°. This value is exact. Explain why.

13. Suppose $\sin n° = \frac{5}{13}$. Find $\cos n°$ and $\tan n°$ without using a table or calculator.

14. According to the table on page 311, sin 1° and tan 1° are both approximately 0.0175. Which is actually larger? How do you know?

15. a. Using the definition of sine, explain why the sine of an acute angle is always less than one.

b. Is the cosine of an acute angle always less than one?

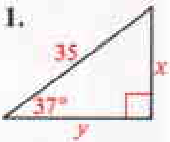
Written Exercises

In these exercises, use a scientific calculator or the table on page 311. Find lengths correct to the nearest integer and angles to the nearest degree.

In Exercises 1–12 find the values of the variables.

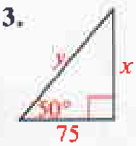
A

1.

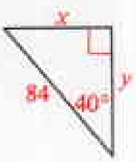


2.

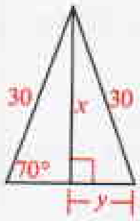
3.



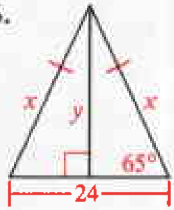
4.



5.



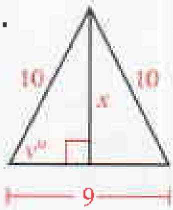
6.



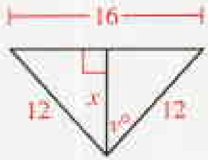
7.

8.

9.

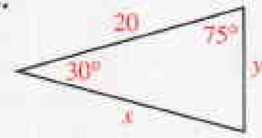


10.



11.

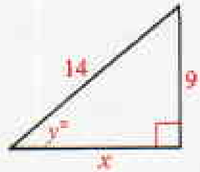
12.



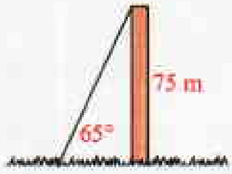
13. a. Use the Pythagorean Theorem to find the value of x in radical form.

b. Use trigonometry to find the values of y, then x.

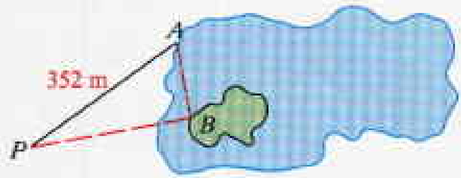
c. Are the values of x from parts (a) and (b) in reasonable agreement?



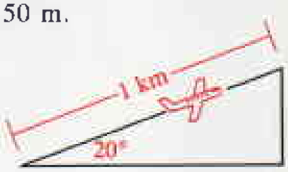
B **14.** A guy wire is attached to the top of a 75 m tower and meets the ground at a 65° angle. How long is the wire?



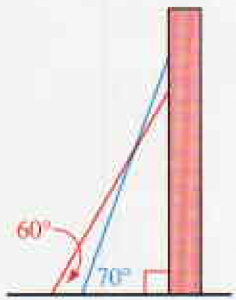
15. To find the distance from point A on the shore of a lake to point B on an island in the lake, surveyors locate point P with $m\angle PAB = 65$ and $m\angle APB = 25$. By measurement, $PA = 352$ m. Find AB.



16. A certain jet is capable of a steady 20° climb. How much altitude does the jet gain when it moves 1 km through the air? Answer to the nearest 50 m.



17. A 6 m ladder reaches higher up a wall when placed at a 70° angle than when placed at a 60° angle. How much higher, to the nearest tenth of a meter?



18. In $\triangle ABC$, $AB = AC = 13$ and $BC = 10$.
  a. Find the length of the altitude from A.
  b. Find the measures of the three angles of $\triangle ABC$.
  c. Find the length of the altitude from C.

19. In $\triangle ABC$, $m\angle B = m\angle C = 72$ and $BC = 10$.
  a. Find AB and AC.
  b. Find the length of the bisector of $\angle A$ to $\overline{BC}$.

20. In $\triangle PAL$, $m\angle A = 90$, $m\angle L = 24$ and median $\overline{AM}$ is 6 cm long. Find PA.

21. The diagonals of rectangle $ABCD$ are 18 cm long and intersect in a 34° angle. Find the length and width of the rectangle.

22. Points A, B, and C are three consecutive vertices of a regular decagon whose sides are 16 cm long. How long is diagonal $\overline{AC}$?

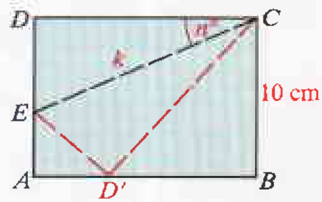
23. Points A, B, C, and D are consecutive vertices of a regular decagon with sides 20 cm long. $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are drawn and intersect at X. Find BX.

For Exercises 24–26 write proofs in paragraph form.

C **24.** Prove that in any triangle with acute angles A and B, $\frac{a}{\sin A} = \frac{b}{\sin B}$. (*Hint*: Draw a perpendicular from the third vertex to $\overline{AB}$. Label it p.)

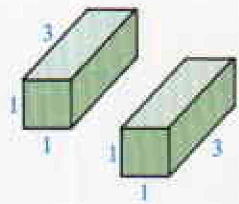
25. Prove: If R is any acute angle, $(\sin R)^2 + (\cos R)^2 = 1$. (*Hint*: From any point on one side of $\angle R$, draw a perpendicular to the other side.)

26. A rectangular card is 10 cm wide. The card is folded so that the vertex D falls at point D' on $\overline{AB}$ as shown. Crease $\overline{CE}$ with length k makes an $n°$ angle with $\overline{CD}$. Prove: $k = \frac{10}{\sin (2n)° \cos n°}$



Challenge

The two blocks of wood have the same size and shape. It is possible to cut a hole in one block in such a way that you can pass the other block completely through the hole. How?



8-7 *Applications of Right Triangle Trigonometry*

Suppose an operator at the top of a lighthouse sights a sailboat on a line that makes a 2° angle with a horizontal line. The angle between the horizontal and the line of sight is called an **angle of depression.** At the same time, a person in the boat must look 2° above the horizontal to see the tip of the lighthouse. This is an **angle of elevation.**

If the top of the lighthouse is 25 m above sea level, the distance x between the boat and the base of the lighthouse can be found in these two ways:

Method 1

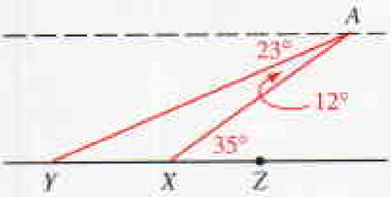
$$\tan 2° = \frac{25}{x}$$
$$x = \frac{25}{\tan 2°}$$
$$x \approx \frac{25}{0.0349}$$
$$x \approx 716.3$$

Method 2

$$\tan 88° = \frac{x}{25}$$
$$x = 25(\tan 88°)$$
$$x \approx 25(28.6363)$$
$$x \approx 715.9$$

Because the tangent values in the table are approximations, the two methods give slightly different answers. In practice, the angle measurement will not be exact, and the boat may be moving. In a case like this we cannot claim high accuracy for our answer. A good answer would be: The boat is roughly 700 m from the lighthouse.

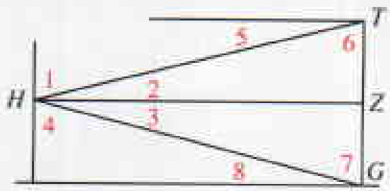
Classroom Exercises



1. Two people at points X and Y sight an airplane at A.
   a. What is the angle of elevation from X to A?
   b. What is the angle of depression from A to X?
   c. What is the angle of depression from A to Y?
   d. What is the angle of elevation from Y to A?
   e. Is the measure of the angle of elevation from Z to A greater or less than 35?

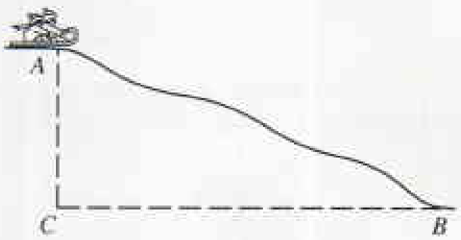
The lines shown are horizontal and vertical lines except for $\overleftrightarrow{HT}$ and $\overleftrightarrow{HG}$. Give the number of the angle and its special name when:

2. A person at *H* sights *T*.
3. A person at *H* sights *G*.
4. A person at *T* sights *H*.
5. A person at *G* sights *H*.



6. A driveway has a 15% grade.
   a. What is the angle of elevation of the driveway?
   b. If the driveway is 12 m long, about how much does it rise?

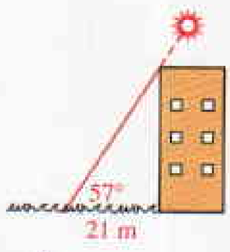
7. A toboggan travels from point *A* at the top of the hill to point *B* at the bottom. Because the steepness of the hill varies, the angle of depression from *A* to *B* is only an approximate measure of the hill's steepness. We can, however, think of this angle of depression as representing the average steepness.
   a. If the toboggan travels 130 m from *A* to *B* and the vertical descent *AC* is 50 m, what is the approximate angle of depression?
   b. Why is your answer approximate?



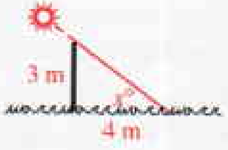
Written Exercises

Express lengths correct to the nearest integer and angles correct to the nearest degree. Use a calculator or the table on page 311.

A 1. When the sun's angle of elevation is 57°, a building casts a shadow 21 m long. How high is the building?



2. At a certain time, a vertical pole 3 m tall casts a 4 m shadow. What is the angle of elevation of the sun?



In Exercises 3–8 first draw a diagram.

3. A kite is flying at an angle of elevation of about 40°. All 80 m of string have been let out. Ignoring the sag in the string, find the height of the kite to the nearest 10 m.

4. An advertising blimp hovers over a stadium at an altitude of 125 m. The pilot sights a tennis court at an 8° angle of depression. Find the ground distance in a straight line between the stadium and the tennis court. (*Note*: In an exercise like this one, an answer saying *about . . . hundred meters* is sensible.)

5. An observer located 3 km from a rocket launch site sees a rocket at an angle of elevation of 38°. How high is the rocket at that moment?

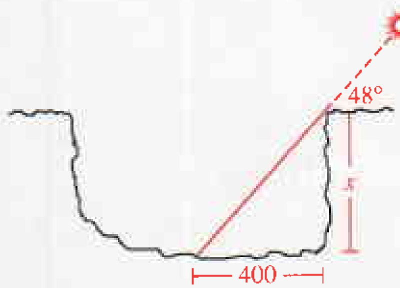
6. To land, an airplane will approach an airport at a 3° angle of depression. If the plane is flying at 30,000 ft, find the ground distance from the airport to the point directly below the plane when the pilot begins descending. Give your answer to the nearest 10,000 feet.

B

7. Martha is 180 cm tall and her daughter Heidi is just 90 cm tall. Who casts the longer shadow, Martha when the sun is 70° above the horizon, or Heidi when the sun is 35° above the horizon? How much longer?

8. Two buildings on opposite sides of a street are 40 m apart. From the top of the taller building, which is 185 m high, the angle of depression to the top of the shorter building is 13°. Find the height of the shorter building.

9. Scientists can estimate the depth of craters on the moon by studying the lengths of their shadows in the craters. Shadows' lengths can be estimated by measuring them on photographs. Find the depth of a crater if the shadow is estimated to be 400 m long and the angle of elevation of the sun is 48°.

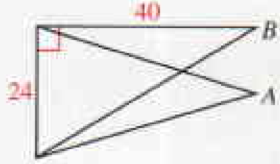


10. A road has a 10% grade.
a. What is the angle of elevation of the road?
b. If the road is 2 km long, how much does it rise?

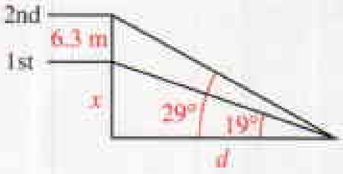
11. A road 1.6 km long rises 400 m. What is the angle of elevation of the road?

12. The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill.
a. With how many pounds of force is gravity pulling on a 3000 lb car on a hill with a 3° angle of elevation?
b. Could you push against the car and keep it from rolling down the hill?

13. A soccer goal is 24 ft wide. Point A is 40 ft in front of the center of the goal. Point B is 40 ft in front of the right goal post.
   a. Which angle is larger, $\angle A$ or $\angle B$?
   b. From which point would you have a better chance of kicking the ball into the goal? Why?



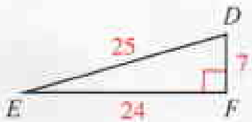
C 14. From the stage of a theater, the angle of elevation of the first balcony is 19°. The angle of elevation of the second balcony, 6.3 m directly above the first, is 29°. How high above stage level is the first balcony? (*Hint*: Use tan 19° and tan 29° to write two equations involving x and d. Solve for d, then find x.)



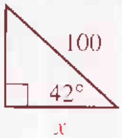
Self-Test 2

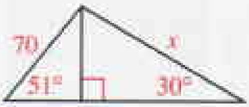
Exercises 1–5 refer to the diagram at the right.

1. $\tan E = \frac{?}{?}$
2. $\cos E = \frac{?}{?}$
3. $\sin E = \frac{?}{?}$
4. $\tan D = \frac{?}{?}$
5. To the nearest integer, $m\angle D =$ _?_.



Find the value of x to the nearest integer.

6.


7.


8.


9. From a point on the ground 100 m from the foot of a cliff, the angle of elevation of the top of the cliff is 24°. How high is the cliff?

Application — *Passive Solar Design*

Passive solar homes are designed to let the sun heat the house during the winter but to prevent the sun from heating the house during the summer. Because the Earth's axis is not perpendicular to the *ecliptic* (the plane of the Earth's orbit around the sun), the sun is lower in the sky in the winter than it is in the summer.

From the latitude of the homesite the architect can determine the elevation angle of the sun (the angle at which a person has to look up from the horizontal to see the sun at noon) during the winter and during the summer. The architect can then design an overhang for windows that will let sunlight in the windows during the winter, but will shade the windows during the summer.

The Earth's axis makes an angle of $23\frac{1}{2}°$ with a perpendicular to the ecliptic plane. So for places in the northern hemisphere between the Tropic of Cancer and the Arctic Circle, the angle of elevation of the sun at noon on the longest day of the year, at the summer solstice, is 90° − the latitude + $23\frac{1}{2}°$. Its angle of elevation at noon on the shortest day, at the winter solstice, is 90° − the latitude − $23\frac{1}{2}°$. For example, in Terre Haute, Indiana, at latitude $39\frac{1}{2}°$ north, the angle of elevation of the sun at noon on the longest day is 74° $(90 - 39\frac{1}{2} + 23\frac{1}{2} = 74)$, and at noon on the shortest day it is 27° $(90 - 39\frac{1}{2} - 23\frac{1}{2} = 27)$.

Exercises

Find the angle of elevation of the sun at noon on the longest day and at noon on the shortest day in the following cities. The approximate north latitudes are in parentheses.

1. Seattle, Washington ($47\frac{1}{2}°$)

2. Chicago, Illinois (42°)

3. Houston, Texas (30°)

4. Los Angeles, California (34°)

5. Nome, Alaska ($64\frac{1}{2}°$)

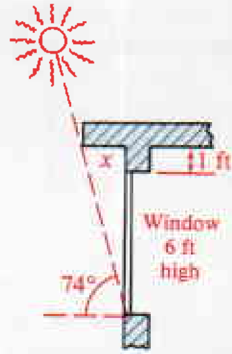
6. Miami, Florida (26°)

7. For a city south of the Tropic of Cancer, such as San Juan, Puerto Rico (18°N), the formula gives a summer solstice angle greater than 90°. What does this mean?

8. For a place north of the Arctic Circle, such as Prudhoe Bay, Alaska (70°N), the formula gives a negative value for the angle of elevation of the sun at noon at the winter solstice. What does this mean?

9. An architect is designing a passive solar house to be located in Terre Haute, Indiana. The diagram shows a cross-section of a wall that will face south. How long must the overhang x be to shade the entire window at noon at the summer solstice?

10. If the overhang has the length found in Exercise 9, how much of the window will be in the sun at noon at the winter solstice?



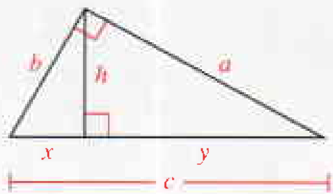
Chapter Summary

1. When $\frac{a}{x} = \frac{x}{b}$, x is the geometric mean between a and b.

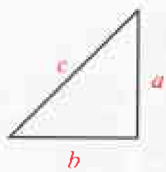
2. A right triangle is shown with the altitude drawn to the hypotenuse.
   a. The two triangles formed are similar to the original triangle and to each other.

   $\frac{x}{h} = \frac{h}{y}$ $\qquad$ $\frac{c}{b} = \frac{b}{x}$ $\qquad$ $\frac{c}{a} = \frac{a}{y}$

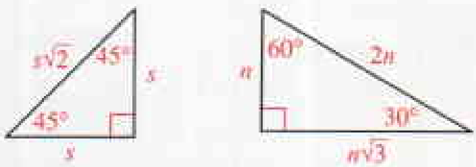
   b. Pythagorean Theorem: $c^2 = a^2 + b^2$



3. The longest side of the triangle shown is c.
   If $c^2 = a^2 + b^2$, then the triangle is a right triangle.
   If $c^2 > a^2 + b^2$, then the triangle is obtuse.
   If $c^2 < a^2 + b^2$, then the triangle is acute.



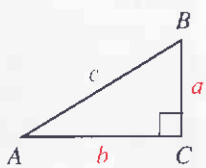
4. The sides of a 45°-45°-90° triangle and the sides of a 30°-60°-90° triangle are related as shown.



5. In the right triangle shown:

   $\tan A = \frac{a}{b}$ $\qquad$ $\sin A = \frac{a}{c}$ $\qquad$ $\cos A = \frac{b}{c}$

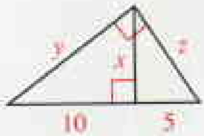
   The tangent, sine, and cosine ratios are useful in solving problems involving right triangles.



Chapter Review

1. Find the geometric mean between 12 and 3. **8–1**

2. x = __?__
3. y = __?__
4. z = __?__



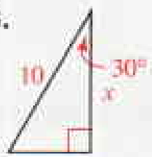
5. The legs of a right triangle are 3 and 6. Find the length of the hypotenuse. **8–2**
6. A rectangle has sides 10 and 8. Find the length of a diagonal.
7. The diagonal of a square has length 14. Find the length of a side.
8. The legs of an isosceles triangle are 10 units long and the altitude to the base is 8 units long. Find the length of the base.

Tell whether a triangle formed with sides having the lengths named is acute, right, or obtuse. If a triangle can't be formed, write *not possible*.

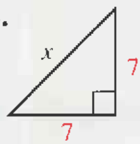
9. 4, 5, 6 — 10. 8, 8, 17 **8–3**
11. 11, 60, 61 — 12. $2\sqrt{3}$, $3\sqrt{2}$, 6

Find the value of *x*. **8–4**

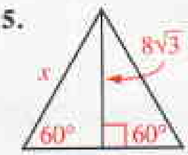
13.



14.



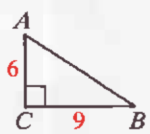
15.



16. The legs of an isosceles right triangle have length 12. Find the lengths of the hypotenuse and the altitude to the hypotenuse.

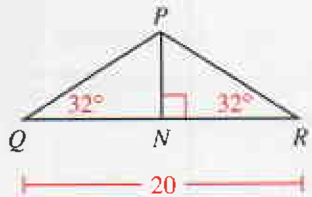
Complete. Find angle measures and lengths correct to the nearest integer. Use a calculator or the table on page 311 if needed. **8–5**

17.



a. tan A = __?__
b. tan B = __?__
c. $m\angle B \approx$ __?__

18.

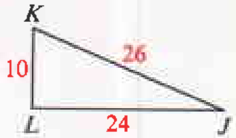


a. QN = __?__
b. $PN \approx$ __?__

Complete. Find angle measures and lengths correct to the nearest integer. Use a calculator or the table on page 311 if needed.

8–6

19.

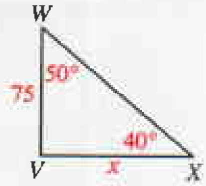


a. $\cos J = \underline{\ ?\ }$

b. $\sin K = \underline{\ ?\ }$

c. $m\angle K \approx \underline{\ ?\ }$

20.



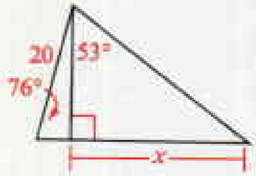
a. $WX \approx \underline{\ ?\ }$

b. $VX \approx \underline{\ ?\ }$

Find the values of *x* and *y* correct to the nearest integer.

21.

22.



23.

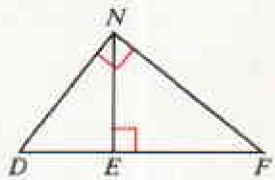
24. Lee, on the ground, looks up at Chong Ye in a hot air balloon at a 35° angle of elevation. If Lee and Chong Ye are 500 ft apart, about how far off the ground is Chong Ye? **8–7**

Chapter Test

Find the geometric mean between the numbers.

1. 5 and 20
2. 6 and 8

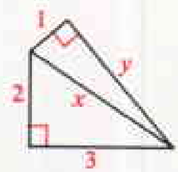
In the diagram, $\angle DNF$ is a right angle and $\overline{NE} \perp \overline{DF}$.



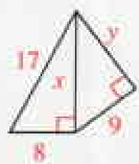
3. $\triangle DNF \sim \triangle\underline{\ ?\ }$, and $\triangle DNF \sim \triangle\underline{\ ?\ }$.
4. NE is the geometric mean between $\underline{\ ?\ }$ and $\underline{\ ?\ }$.
5. NF is the geometric mean between $\underline{\ ?\ }$ and $\underline{\ ?\ }$.
6. If $DE = 10$ and $EF = 15$, then $ND = \underline{\ ?\ }$.

Find the values of *x* and *y*.

7.



8.



Tell whether a triangle formed with sides having the lengths named is acute, right, or obtuse. If a triangle can't be formed, write *not possible*.

9. 3, 4, 8

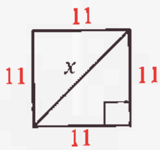
10. 11, 12, 13

11. 7, 7, 10

12. $\frac{3}{5}, \frac{4}{5}, 1$

Find the value of *x*.

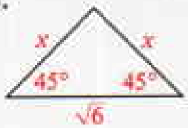
13.



14.

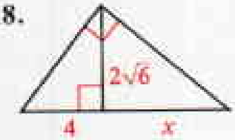
15.

16.



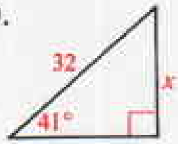
17.

18.

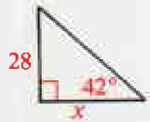


Find lengths correct to the nearest integer and angles correct to the nearest degree.

19.

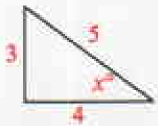


20.

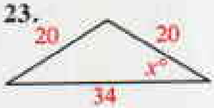


21.

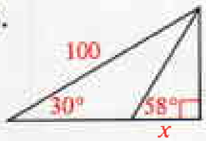
22.



23.

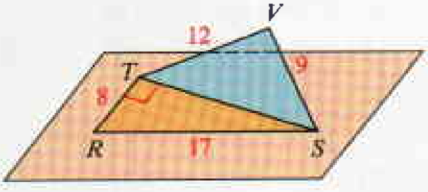


24.



25. The sides of a rhombus are 4 units long and one diagonal has length 4. How long is the other diagonal?

26. In the diagram, $\angle RTS$ is a right angle; $\overline{RT}$, $\overline{RS}$, $\overline{VT}$ and $\overline{VS}$ have the lengths shown.
What is the measure of $\angle V$? Explain.



27. From the top of a lighthouse 18 m high, the angle of depression to sight a boat is 4°. What is the distance between the boat and the base of the lighthouse?

Preparing for College Entrance Exams

Strategy for Success

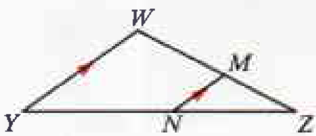
Problems in college entrance exams often involve right triangles. One thing you can do in preparing for the exams is to learn the common right-triangle lengths listed on page 295. These Pythagorean triples are often used on tests where calculators are not allowed. Also, keep in mind that if a, b, and c are the lengths of the sides of a right triangle, then for any $k > 0$, ak, bk, and ck are also lengths of sides of a right triangle.

Indicate the best answer by writing the appropriate letter.

1. In $\triangle ABC$, $m\angle A : m\angle B : m\angle C = 2:5:5$. $m\angle B =$
(A) 75 **(B)** 60 **(C)** 30 **(D)** 40 **(E)** 100

2. The proportion $\frac{t}{z} = \frac{m}{k}$ is *not* equivalent to:
(A) $\frac{t - z}{z} = \frac{m - k}{k}$ **(B)** $\frac{k}{z} = \frac{m}{t}$ **(C)** $\frac{t}{m} = \frac{k}{z}$ **(D)** $tk = mz$ **(E)** $\frac{z}{t} = \frac{k}{m}$

3. If $\triangle ABC \sim \triangle DEF$, which statement is not necessarily true?
(A) $\angle C \cong \angle F$ **(B)** $\overline{BC} \cong \overline{EF}$ **(C)** $\frac{AB}{BC} = \frac{DE}{EF}$
(D) $m\angle A + m\angle E = m\angle B + m\angle D$ **(E)** $AC \cdot DE = DF \cdot AB$



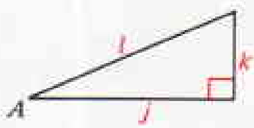
4. If $ZY = 2x + 9$, $ZM = 10$, $ZN = x + 3$, and $MW = x$, then $x =$
(A) $2 + \sqrt{34}$ **(B)** -12 **(C)** 12 **(D)** 5 **(E)** -5

5. $\overrightarrow{BD}$ bisects $\angle ABC$ and D lies on $\overline{AC}$. If $AB = 6$, $BC = 14$, and $AC = 14$, find AD.
(A) 6 **(B)** 8.4 **(C)** 9.8 **(D)** 7 **(E)** 4.2

6. Find the geometric mean of $2x$ and $2y$.
(A) $2\sqrt{xy}$ **(B)** $\sqrt{2xy}$ **(C)** $2\sqrt{x + y}$ **(D)** $\sqrt{2(x + y)}$ **(E)** $4xy$

7. If $XY = 8$, $YZ = 40$, and $XZ = 41$, then:
(A) $\triangle XYZ$ is acute **(B)** $\triangle XYZ$ is right **(C)** $\triangle XYZ$ is obtuse
(D) $m\angle Y < m\angle Z$ **(E)** no $\triangle XYZ$ is possible

8. A rhombus contains a 120° angle. Find the ratio of the length of the longer diagonal to the length of the shorter diagonal.
(A) $\sqrt{3}:1$ **(B)** $\sqrt{3}:3$ **(C)** $\sqrt{2}:1$ **(D)** $\sqrt{2}:2$ **(E)** cannot be determined



9. $k =$
(A) $j \sin A$ **(B)** $j \tan A$ **(C)** $\frac{l}{\sin A}$
(D) $l \cos A$ **(E)** $l \tan A$

10. The legs of an isosceles triangle have length 4 and the base angles have measure 65. If $\sin 65° \approx 0.91$, $\cos 65° \approx 0.42$, and $\tan 65° \approx 2.14$, then the approximate length of the base of the triangle is:
(A) 1.7 **(B)** 1.9 **(C)** 3.4 **(D)** 3.6 **(E)** 4.4

Cumulative Review: Chapters 1–8

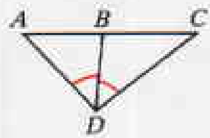
In Exercises 1–8, complete each statement.

A

1. If S is between R and T, then $RS + ST = RT$ by the $\underline{\ ?\ }$.
2. A statement that is accepted without proof is called a $\underline{\ ?\ }$.
3. A statement that can be proved easily by using a theorem is called a $\underline{\ ?\ }$.
4. To write an indirect proof, you assume temporarily that the $\underline{\ ?\ }$ is not true.
5. A conditional and its $\underline{\ ?\ }$ are always logically equivalent.
6. The sides of an obtuse triangle have lengths x, $2x + 2$, and $2x + 3$. $\underline{\ ?\ } < x < \underline{\ ?\ }$.
7. In an isosceles right triangle, the ratio of the length of a leg to the length of the hypotenuse is $\underline{\ ?\ }$.
8. If $\sin B = \frac{8}{17}$, then $\cos B = \underline{\ ?\ }$.
9. Given: A triangle is equiangular only if it is isosceles.
   a. Write an if-then statement that is logically equivalent to the given conditional.
   b. State the converse. Sketch a diagram to disprove the converse.
10. Use inductive reasoning to guess the next two numbers in the sequence:
    1, 2, 6, 15, 31, 56, . . .
11. When two parallel lines are cut by a transversal, two corresponding angles have measures x^2 and $6x$. Find the measure of each angle.

B

12. In $\triangle XYZ$, $m\angle X : m\angle Y : m\angle Z = 3:3:4$.
    a. Is $\triangle XYZ$ scalene, isosceles, or equilateral?
    b. Is $\triangle XYZ$ acute, right, or obtuse?
    c. Name the longest side of $\triangle XYZ$.
13. If $AB = x - 5$, $BC = x - 2$, $CD = x + 4$, and $DA = x$, find the value of x.



14. The diagonals of a rhombus have lengths 18 and 24. Find the length of one side.
15. Write a paragraph proof: If $\overline{AX}$ is a median and an altitude of $\triangle ABC$, then $\triangle ABC$ is isosceles.
16. Given: $NPQRST$ is a regular hexagon.
    Prove: $NPRS$ is a rectangle.
    (Begin by drawing a diagram.)
17. Given: $\angle WXY \cong \angle XZY$
    Prove: $(XY)^2 = WY \cdot ZY$